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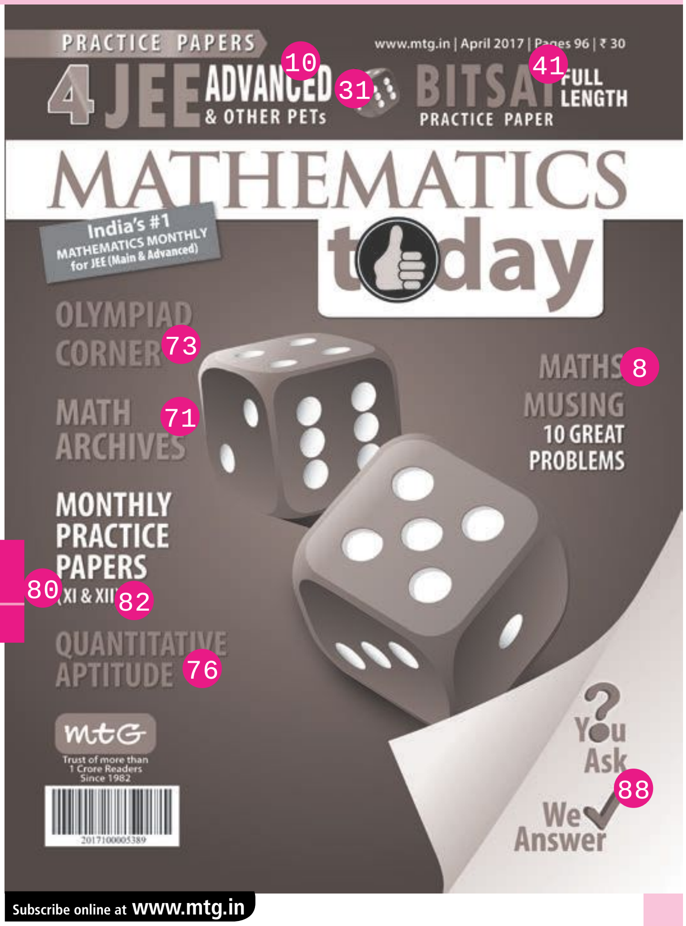
Corporate Office:
Plot 99, Sector 44 Institutional Area,
Gurgaon -122 003 (HR), Tel : 0124-6601200
e-mail : info@mtg.in website : www.mtg.in

Regd. Office:
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
Managing Editor : Mahabir Singh
Editor : Anil Ahlawat



CONTENTS

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MATHS MUSING

Maths Musing was started in January 2003 issue of Mathematics Today. The aim of Maths Musing is to augment the chances of bright students seeking admission into IITs with additional study material.

During the last 10 years there have been several changes in JEE pattern. To suit these changes Maths Musing also adopted the new pattern by changing the style of problems. Some of the Maths Musing problems have been adapted in JEE benefitting thousand of our readers. It is heartening that we receive solutions of Maths Musing problems from all over India.

Maths Musing has been receiving tremendous response from candidates preparing for JEE and teachers coaching them. We do hope that students will continue to use Maths Musing to boost up their ranks in JEE Main and Advanced.

PROBLEM Set 172

JEE MAIN

1. The lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular. Then

(a) $\frac{a}{a'}+\frac{c}{c'}=-1$ (b) $\frac{a}{a'}+\frac{c}{c'}=1$

(c) $aa' + cc' = -1$ (d) $aa' + cc' = 1$

2. In an ellipse the distance between the foci is 8 and the distance between the directrices is 10. The angle α that the latusrectum subtends at the centre is

(a) $\cos^{-1}\frac{17}{19}$ (b) $\cos^{-1}\frac{19}{21}$

(c) $\cos^{-1}\frac{17}{23}$ (d) $\tan^{-1}\frac{17}{19}$

3. If $2a + 3b + 6c = 0$, $a, b, c \in R$, then the equation $ax^2 + bx + c = 0$ has a root in

(a) (0, 1) (b) (2, 3)

(c) (4, 5) (d) none of these

4. If 3 distinct numbers are chosen randomly from the first 100 natural numbers, then the probability that all three of them are divisible by both 2 and 3, is

(a) $\frac{4}{55}$ (b) $\frac{4}{35}$ (c) $\frac{4}{33}$ (d) $\frac{4}{1155}$

5. If $x^3\frac{dy}{dx}=y^3+y^2\sqrt{y^2-x^2}$ and $y = 1$ when $x = 1$, then $y = 2$ when $x =$

(a) $\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $\frac{6}{5}$ (d) $\frac{8}{5}$

JEE ADVANCED

6. Let $a > 0$ the equation $a\cdot 2^{\sin^2 x}+a\cdot 2^{-\sin^2 x}-2=0$ has a root if $a \in$

(a) (1, 2) (b) $\left(\frac{1}{2},1\right)$ (c) $\left[\frac{3}{5},1\right]$ (d) $\left[\frac{4}{5},1\right]$

COMPREHENSION

Let ABC be a triangle with inradius r and circumradius R. Its incircle touches the sides BC, CA, AB at A_1, B_1, C_1 respectively. The incircle of triangle $A_1B_1C_1$ touches its sides at A_2, B_2, C_2 respectively and so on.

7. In triangle $A_4B_4C_4$, the value of A_4 is

(a) $\frac{3\pi+A}{8}$ (b) $\frac{5\pi-A}{8}$ (c) $\frac{5\pi-A}{16}$ (d) $\frac{5\pi+A}{16}$

8. $\frac{B_1C_1}{BC}=$

(a) $\sin\frac{B}{2}\sin\frac{C}{2}$ (b) $\cos\frac{B}{2}\cos\frac{C}{2}$

(c) $\cos\left(\frac{B-C}{2}\right)-\sin\frac{A}{2}$

(d) $\sin\frac{A}{2}+\cos\left(\frac{B-C}{2}\right)$

INTEGER MATCH

9. The sum of three numbers in G.P. is 42. If each of the extremes be multiplied by 4 and the mean by 5, then the products are in A.P. The least of the original numbers is

MATRIX MATCH

10. Match the following.

	List-I		List-II
P.	If $y=\cos\left(\frac{1}{3}\sin^{-1}x\right)$, then $(1-x^2)y_2-xy_1+\lambda y=0$ where $\lambda =$	1.	$\frac{7}{4}$
Q.	If the function $y=\frac{ax+b}{(4-x)(x-1)}$ has an extreme value at (2, 1), then its maximum value is	2.	$\frac{5}{2}$
R.	The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at (–2, 0) and cuts the y-axis where its gradient is 3. Then $a + b + c =$	3.	$\frac{1}{9}$
S.	The product of the abscissae of the points on the curve $y = x^3 - 7x^2 + 6x + 5$ at which tangents pass through the origin is	4.	$-\frac{1}{9}$

	P	Q	R	S
(a)	3	3	1	2
(b)	1	2	3	4
(c)	4	3	2	1
(d)	3	3	2	1

See Solution Set of Maths Musing 171 on page no. 89

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Thomas Jacob

Total selections in C.G.
34

Exam on 21st May

JEE ADVANCED

PRACTICE PAPER 2017

PAPER-I

SECTION-1

SINGLE CORRECT ANSWER TYPE

1. A committee of 6 is chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. The number of ways this can be done, if two particular women refuse to serve on the same committee, is
(a) 8000 (b) 7800 (c) 7600 (d) 7200

2. If $\ln_{\sin x} \cos x + \ln_{\cos x} \sin x = 2$, then $x =$
(a) $\frac{n\pi}{4}$ (b) $(n+1)\frac{\pi}{4}$
(c) $(2n+1)\frac{\pi}{4}$ (d) $(4n+1)\frac{\pi}{4}$

3. The slope of the straight line which is both tangent and normal to the curve $4x^3 = 27y^2$ is
(a) ± 1 (b) $\pm\frac{1}{2}$ (c) $\pm\frac{1}{\sqrt{2}}$ (d) $\pm\sqrt{2}$

4. If the sum of the roots of the equation $ax^2 + bx + c = 0$ is equal to the sum of the reciprocals of their squares, then $\frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in
(a) A.P. (b) G.P.
(c) H.P. (d) none of these

5. A person goes to office either by car, scooter, bus or train the probabilities of which being $\frac{1}{7}, \frac{3}{7}, \frac{2}{7}, \frac{1}{7}$ respectively. The probability that he reaches office late, if he takes car, scooter, bus or train is $\frac{2}{9}, \frac{1}{9}, \frac{4}{9}, \frac{1}{9}$ respectively. If he reached office in time, the probability that he travelled by car is
(a) $\frac{1}{5}$ (b) $\frac{1}{9}$
(c) $\frac{2}{11}$ (d) $\frac{1}{7}$

SECTION-2

MORE THAN ONE CORRECT ANSWER TYPE

6. Let $f(x) = \ln|x|$ and $g(x) = \sin x$. If A is the range of $f(g(x))$ and B is the range of $g(f(x))$, then
(a) $A \cup B = (-\infty, 1]$ (b) $A \cup B = (-\infty, \infty)$
(c) $A \cap B = [-1, 0]$ (d) $A \cap B = [0, 1]$

7. Let $y = f(x)$ be a curve in the first quadrant such that the triangle formed by the co-ordinate axes and the tangent at any point on the curve has area 2. If $y(1) = 1$, then $y(2) =$
(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

8. If $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ then $\theta =$
(a) $\frac{7\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{11\pi}{24}$ (d) $\frac{\pi}{24}$

9. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$ be the ends of the latusrectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latusrectum PQ are
(a) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$
(b) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$
(c) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$
(d) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

10. The equation of the plane containing the line $2x - y + z - 3 = 0$, $3x + y + z = 5$ and at a distance of $\frac{1}{\sqrt{6}}$ from the point $(2, 1, -1)$ is
(a) $x + y + z - 3 = 0$ (b) $2x - y - z - 3 = 0$
(c) $2x - y + z - 3 = 0$ (d) $62x + 29y + 19z - 105 = 0$

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CAMPUS: Village Talegaon, Near Sindhudurg Rly. Stn., Post Oras, Tal. Malvan, Dist. Sindhudurg, Maharashtra | Tel.: 02362-228550-52

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11. Tangents are drawn from a point P on the circle $C : x^2 + y^2 = a^2$ to the circle $C_1 : x^2 + y^2 = b^2$. If these tangents cut the circle C at Q and R and if QR is a tangent to the circle C_1, then the area of triangle PQR is

(a) $3\sqrt{3}b^2$ (b) $\frac{3\sqrt{3}}{4}a^2$

(c) $\frac{3\sqrt{3}}{2}ab$ (d) $2\sqrt{3}ab$

12. The internal bisector of $\angle A$ of triangle ABC meets side BC at D. A line drawn through D perpendicular to AD meets the side AC at E and the side AB at F. If a, b, c are the sides of triangle ABC, then

(a) AE is H.M. of b and c

(b) $AD = \frac{2bc}{b+c}\cos\frac{A}{2}$ (c) $EF = \frac{4bc}{b+c}\sin\frac{A}{2}$

(d) triangle AEF is isosceles.

13. Let a, b, c, d, e be five numbers such that a, b, c are in A.P., b, c, d are in G.P. and c, d, e are in H.P. If $a = 2$ and $e = 18$, then $b =$

(a) 2 (b) – 2 (c) 4 (d) – 4

SECTION-3

INTEGER ANSWER TYPE

14. If $n \in N$, then the highest integer m such that 2^m divides $3^{2n+2} - 8n - 9$ is

15. If $\lim_{x\to 0}\left(1 + x + \frac{f(x)}{x}\right)^{1/x} = e^3$, then $\lim_{x\to 0}\frac{f(x)}{x^2} =$

16. Let $px^4 + qx^3 + rx^2 + sx + t = \begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$

be an identity, where p, q, r, s, t are constants. Then $t =$

17. Let $n \in N$, $n \le 5$. If $I_n = \int_0^1 e^x (x-1)^n dx = 16 - 6e$, then $n =$

18. If ω is complex cube root of unity and a, b, c are such that $\frac{1}{a+\omega} + \frac{1}{b+\omega} + \frac{1}{c+\omega} = 2\omega^2$

and $\frac{1}{a+\omega^2} + \frac{1}{b+\omega^2} + \frac{1}{c+\omega^2} = 2\omega$,

then $\frac{1}{a+1} + \frac{1}{b+1} + \frac{1}{c+1} =$

PAPER -II

SECTION-1

SINGLE CORRECT ANSWER TYPE

1. A plane moves such that the volume of the tetrahedron formed with coordinate planes is a constant k. The locus of the foot of perpendicular from the origin O on it is $(x^2 + y^2 + z^2)^3 =$

(a) 12 $kxyz$ (b) 24 $kxyz$

(c) 48 $kxyz$ (d) 6 $kxyz$

2. An open box with a square base is to be made from 400 m^2 of lumber. When its volume is maximum, the ratio of an edge of the base to the height is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\sqrt{2}$

3. $S = \sum_{r=1}^{n-1} \cos^2\frac{r\pi}{n} =$

(a) $\frac{n}{2}$ (b) $\frac{n-1}{2}$ (c) $\frac{n-2}{2}$ (d) $\frac{n+1}{2}$

4. Let S be the sum, P be the product and R be the sum of reciprocals of n terms of a G.P. Then

(a) $R = S \cdot P^{1/n}$ (b) $R = S \cdot P^{2/n}$

(c) $S = R \cdot P^{1/n}$ (d) $S = R \cdot P^{2/n}$

5. Let $f(x)$ be a function such that $f'(x) = f(x), f(0) = 1$ and $g(x)$ be a function such that $f(x) + g(x) = x^2$. Then $\int_0^1 f(x)g(x)dx =$

(a) $e - \frac{e^2}{2} - \frac{5}{2}$ (b) $e + \frac{e^2}{2} - \frac{3}{2}$

(c) $e - \frac{e^2}{2} - \frac{3}{2}$ (d) $e + \frac{e^2}{2} + \frac{5}{2}$

6. If the system of equations $x + ky + 3z = 0$, $3x + ky - 2z = 0$, $2x + 3y - 4z = 0$ has non trivial solution, then $\frac{xy}{z^2} =$

(a) $\frac{5}{6}$ (b) $\frac{-5}{6}$ (c) $\frac{6}{5}$ (d) $\frac{-6}{5}$

SECTION-2

MORE THAN ONE CORRECT ANSWER TYPE

7. If $x + |y| = 2y$, then y as a function of x is

(a) defined for all x

(b) continuous at $x = 0$

(c) differentiable for all x

(d) $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

8. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors, equally inclined to each other at angle θ, where $\frac{\pi}{3} \le \theta \le \frac{\pi}{2}$. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the vertices of a triangle and $\vec{g}$ is the position vector of its centroid, then

(a) $|\vec{g}| \ge \frac{1}{\sqrt{3}}$ (b) $|\vec{g}| \le 1$

(c) $|\vec{g}| \le \sqrt{\frac{2}{3}}$ (d) $|\vec{g}| \ge 1$

9. Let z_1 and z_2 be two distinct complex numbers and let $z = (1 - t) z_1 + tz_2$, for some real number t with $0 < t < 1$, then

(a) $|z - z_1| + |z - z_2| = |z_1 - z_2|$

(b) $\arg (z - z_1) = \arg |z - z_2|$

(c) $\arg (z - z_1) = \arg (z_2 - z_1)$

(d) $\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

10. If a hyperbola passes through a focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, and the product of their eccentricities is 1, then

(a) the hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(b) the hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

(c) a focus of hyperbola is (5, 0)

(d) a focus of hyperbola is $(5\sqrt{3}, 0)$

11. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2 + kn + k^2}$

and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$. Then

(a) $S_n < \frac{\pi}{3\sqrt{3}}$ (b) $S_n > \frac{\pi}{3\sqrt{3}}$

(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

12. The area of the triangle formed by the tangent to the curve $y = x^2 + bx - b$ at the point (1, 1) and the coordinate axes is 2. Then $b =$

(a) 3 (b) –3

(c) $1 + 2\sqrt{2}$ (d) $1 - 2\sqrt{2}$

13. If the equations $x + y = 1$, $(c + 2) x + (c + 4) y = 6$, $(c + 2)^2 x + (c + 4)^2 y = 36$ are consistent, then $c =$

(a) 1 (b) 2 (c) 3 (d) 4

14. If the solution of $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = x$, $y(0) = y(1) = 1$ is given by $y^2 = f(x)$ then

(a) $f(x)$ is monotonically increasing $\forall\, x \in (1, \infty)$

(b) $f(x) = 0$ has only one root

(c) $f(x)$ is neither even nor odd

(d) $f(x)$ has 3 real roots

SECTION-3

COMPREHENSION TYPE

Paragraph for Q. No. 15 and 16

For $x \in \left(0, \frac{\pi}{4}\right)$,

let $S_n = \sum_{r=1}^{2n} \sin(\sin^{-1} x^{3r-2})$, $C_n = \sum_{r=1}^{2n} \cos(\cos^{-1} x^{3r-1})$

and $T_n = \sum_{r=1}^{2n} \tan(\tan^{-1} x^{3r})$,

where $n \in N$ and $n \ge 3$

15. The correct order of S_n, C_n and T_n is given by

(a) $S_n > T_n > C_n$ (b) $S_n < C_n < T_n$

(c) $S_n < T_n < C_n$ (d) $S_n > C_n > T_n$

16. The value of 'x' for which $S_n = C_n + T_n$, is

(a) $\sin\left(\frac{\pi}{5}\right)$ (b) $2\sin\left(\frac{\pi}{5}\right)$

(c) $2\sin\left(\frac{\pi}{10}\right)$ (d) $\sin\left(\frac{\pi}{10}\right)$

Paragraph for Q. No. 17 and 18

The normal at any point (x_1, y_1) of curve is line perpendicular to tangent at (x_1, y_1). In case of parabola $y^2 = 4ax$ the equation of normal is $y = mx - 2am - am^2$ (m is slope of normal). In case of rectangular hyperbola $xy = c^2$ the equation of normal at $\left(ct, \frac{c}{t}\right)$ is $xt^3 - yt - ct^4 + c = 0$ and the shortest distance between any two curve always exist along the common normal.

17. If normal at (5, 3) of hyperbola $xy - y - 2x - 2 = 0$ intersects the curve again at $(\alpha, \beta - 29)$, then $\frac{\beta}{\alpha}$ is

(a) 10 (b) 20 (c) 30 (d) 40

18. If the shortest distance between $2y^2 - 2x + 1 = 0$ and $2x^2 - 2y + 1 = 0$ is d then the number of solutions of $|\sin\alpha| = 2\sqrt{2}d\ (\alpha \in [-\pi, 2\pi])$ is

(a) 3 (b) 4

(c) 5 (d) none of these

SOLUTIONS

PAPER - I

1. (b) : Let the men be $M_1, M_2, ..., M_{10}$ and women be $W_1, W_2, ..., W_7$. Let W_1 and W_2 do not want to be on the same committee.

The six member committee can contain 4 men and 2 women or 3 men and 3 women.

The number of ways of forming $4M$, $2W$ committee is

$$\binom{10}{4}\cdot\left[\binom{5}{2}+2\binom{5}{1}\right] = 4200 \quad ...(1)$$

$\binom{5}{2}$ is the number of ways without W_1 and W_2

$\binom{5}{1}$ is the number of ways with W_1, and without W_2, or with W_2 and without W_1. The number of ways of forming $3M$, $3W$ committee is

$$\binom{10}{3}\cdot\left[\binom{5}{3}+2\binom{5}{2}\right] = 3600 \quad ...(2)$$

$\binom{5}{3}$ is the number of ways without W_1 and W_2. $\binom{5}{2}$ is the number of ways with W_1 or W_2 but not both.

$\therefore$ The desired number is 4200 + 3600 = 7800.

2. (d) : $\dfrac{\ln\cos x}{\ln\sin x} + \dfrac{\ln\sin x}{\ln\cos x} = 2$

$t + \dfrac{1}{t} = 2 \Rightarrow t = 1$

$\therefore \dfrac{\ln\sin x}{\ln\cos x} = 1 \Rightarrow \ln\sin x - \ln\cos x = 0$

$\therefore \ln\tan x = 0 \Rightarrow \tan x = 1 = \tan\dfrac{\pi}{4}$

$\therefore x = n\pi + \dfrac{\pi}{4} = (4n+1)\dfrac{\pi}{4}$.

3. (d) : $x = 3t^2, y = 2t^3, \dfrac{dy}{dx} = t$

The tangent at t, $y - 2t^3 = t(x - 3t^2)$, $tx - y = t^3$...(1)

The normal at t_1, $t_1 y + x = 2t_1^4 + 3t_1^2$ ...(2)

(1), (2) are identical, $\dfrac{t}{1} = \dfrac{-1}{t_1} = \dfrac{t^3}{2t_1^4 + 3t_1^2}$

$\Rightarrow -t^3 = 2t_1^3 + 3t_1,\ t_1 = -\dfrac{1}{t}$.

Eliminating t_1, we get $t^6 = 2 + 3t^2$

$\Rightarrow t^2 = 2,\ t = \pm\sqrt{2}$.

The lines are $y = \pm\sqrt{2}(x-2)$.

4. (c): $\alpha+\beta = \dfrac{-b}{a}, \alpha\beta = \dfrac{c}{a}, \alpha+\beta = \dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}$

$\Rightarrow \alpha+\beta = \dfrac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \dfrac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$

$\therefore (\alpha+\beta)(\alpha\beta)^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$-\dfrac{b}{a}\cdot\dfrac{c^2}{a^2} = \dfrac{b^2}{a^2} - \dfrac{2c}{a} = \dfrac{b^2 - 2ca}{a^2}$

$\Rightarrow -bc^2 = ab^2 - 2ca^2 \Rightarrow ab^2 + bc^2 = 2ca^2$

$\Rightarrow ab^2, ca^2, bc^2$ are in A.P.

Dividing by abc, we get $\dfrac{b}{c}, \dfrac{a}{b}, \dfrac{c}{a}$ are in A.P. $\dfrac{c}{a}, \dfrac{a}{b}, \dfrac{b}{c}$ are in A.P., $\dfrac{a}{c}, \dfrac{b}{a}, \dfrac{c}{b}$ are in H.P.

5. (d) : Let C, S, B, T be the events of the person going by car, scooter, bus, train respectively.

$P(C) = \dfrac{1}{7}, P(S) = \dfrac{3}{7}, P(B) = \dfrac{2}{7}, P(T) = \dfrac{1}{7}$.

Let $\bar{L}$ be the event that the person reaching the office in time

$P(\bar{L}|C) = \dfrac{7}{9}, P(\bar{L}|S) = \dfrac{8}{9}, P(\bar{L}|B) = \dfrac{5}{9}, P(\bar{L}|T) = \dfrac{8}{9}$

$$\therefore P(C|\bar{L}) = \frac{P(C)P(\bar{L}|C)}{P(\bar{L})} = \frac{\frac{1}{7}\cdot\frac{7}{9}}{\frac{1}{7}\cdot\frac{7}{9}+\frac{3}{7}\cdot\frac{8}{9}+\frac{2}{7}\cdot\frac{5}{9}+\frac{1}{7}\cdot\frac{8}{9}}$$

$$= \frac{7}{49} = \frac{1}{7}$$

6. (a, c) : The range of $|\sin x| = [0, 1]$.

A = range of $\ln|\sin x| = (-\infty, 0]$.

The range of $\ln|x| = (-\infty, \infty)$.

B = range of $\sin\ln|x| = [-1, 1]$

$\therefore A \cup B = (-\infty, 0] \cup [-1, 1] = (-\infty, 1]$,

$A \cap B = (-\infty, 0) \cap [-1, 1] = [-1, 0]$.

7. (a, d) : The tangent at $P(x, y)$ is $Y - y - (X - x)y_1 = 0$

It meets x-axis at $A\left(x - \dfrac{y}{y_1}, 0\right)$ and y-axis at $B(0, y - xy_1)$.

$\Delta OAB = 2 \Rightarrow OA \cdot OB = 4$

$$\left(x - \frac{y}{p}\right)(y - xp) = 4,\ p = \frac{dy}{dx},\ (y - xp)^2 = -4p$$

$$y - xp = 2\sqrt{-p} \quad \Rightarrow \quad y = xp + 2\sqrt{-p}$$

The general solution is $y = cx + 2\sqrt{-c}$ ...(1)

$x = 1, y = 1 \Rightarrow c = -1, y = 2 - x, y(2) = 0$

Differentiating (1) w.r.t c, $0 = x - \frac{1}{\sqrt{-c}}$ ...(2)

Eliminating c from (1), using (2)

$$y = -\frac{1}{x} + \frac{2}{x} = \frac{1}{x},\ y(2) = \frac{1}{2}$$

8. (a, c) : Subtracting R_3 from R_1 and R_2,

$$\begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1 + 4\sin 4\theta \end{vmatrix} = 0$$

$\sin^2\theta + \cos^2\theta + 1 + 4\sin 4\theta = 0$

$$\Rightarrow \quad \sin 4\theta = -\frac{1}{2} \quad \Rightarrow \quad \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$$

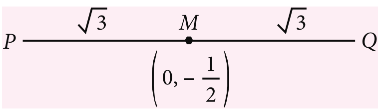
9. (b, c) : $\frac{x^2}{4} + \frac{y^2}{1} = 1,\ a = 2,\ b = 1$

The ends of latusrectum PQ are

$$\left(\pm\sqrt{a^2 - b^2}, -\frac{b^2}{a}\right)$$

$$P = \left(-\sqrt{3}, -\frac{1}{2}\right), Q\left(\sqrt{3}, -\frac{1}{2}\right)$$

The midpoint of PQ is $M\left(0, -\frac{1}{2}\right)$ which is the focus of the parabola.



The vertex of the parabola is $V\left(0, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\right)$.

$\therefore$ The parabolas are $x^2 = \mp 2\sqrt{3}\left(y - \left(-\frac{1}{2} \pm \frac{\sqrt{3}}{2}\right)\right)$

i.e., $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$ and $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$.

10. (c, d) : The plane is

$2x - y + z - 3 + \lambda$

$(3x + y + z - 5) = 0$ ...(i)

or $(2 + 3\lambda)x + (\lambda - 1)y + (\lambda + 1)z - (5\lambda + 3) = 0$

Its distance from $(2, 1, -1)$ is $\frac{1}{\sqrt{6}}$

$$\frac{|4 + 6\lambda + \lambda - 1 - \lambda - 1 - 5\lambda - 3|}{\sqrt{(2 + 3\lambda)^2 + (\lambda - 1)^2 + (\lambda + 1)^2}} = \frac{1}{\sqrt{6}}$$

$\Rightarrow 6(\lambda - 1)^2 = 11\lambda^2 + 12\lambda + 6 \quad \Rightarrow \quad (5\lambda + 24)\lambda = 0$

$\lambda = 0$, (i) $\Rightarrow 2x - y + z - 3 = 0$

$\lambda = \frac{-24}{5}$, (i)

$\Rightarrow 5(2x - y + z - 3) - 24(3x + y + z - 5) = 0$

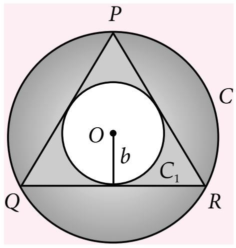
$\Rightarrow 62x + 29y + 19z - 105 = 0.$

11. (a, b, c) : All the sides of ΔPQR touch C_1.

$\therefore$ C_1 is incircle and C is the circumcircle with same centre.

The triangle is equilateral.

$R = 2r \Rightarrow a = 2b$



$$\Delta PQR = \frac{\sqrt{3}}{4}PR^2$$

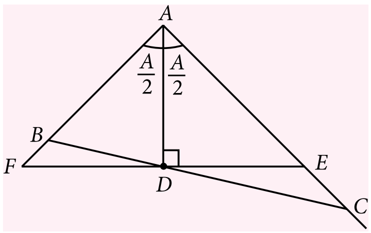
But $3b = \frac{\sqrt{3}}{2}PR$

$$\Delta PQR = \frac{\sqrt{3}}{4} \cdot 4 \cdot 3b^2 = 3\sqrt{3}b^2$$

$$= \frac{3}{2}\sqrt{3}\,ab = \frac{3\sqrt{3}}{4}a^2 \text{ since } a = 2b.$$

12. (a, b, c, d) : $\Delta ABC = \Delta ABD + \Delta ADC$

$$\frac{1}{2} \cdot bc\sin A = \frac{1}{2}c \cdot AD\sin\frac{A}{2} + \frac{1}{2}b \cdot AD\sin\frac{A}{2}$$



$$bc \cdot 2\cos\frac{A}{2} = (b + c)AD$$

$\therefore \quad AD = \frac{2bc}{b + c}\cos\frac{A}{2}$ ...(1)

$$AE\cos\frac{A}{2} = AD \Rightarrow AE = \frac{2bc}{b + c}$$

$\therefore \quad AE$ is H.M. of b and c ...(2)

$EF = 2ED = 2AD\tan\frac{A}{2} = \frac{4bc}{b + c}\sin\frac{A}{2}$, using (1)

$AD \perp EF$. AD is angle bisector

$\therefore$ AEF is isosceles, from (2)

13. (b, c) : a, b, c are in A.P. $\Rightarrow a + c = 2b$ ...(1)

b, c, d are in G.P. $\Rightarrow bd = c^2$ ...(2)

c, d, e are in H.P. $\Rightarrow d = \dfrac{2ce}{c+e}$ ...(3)

Since $a = 2,\ e = 18,\ b = \dfrac{2+c}{2},\ d = \dfrac{36c}{c+18}$.

Now (2) $\Rightarrow \dfrac{18c(2+c)}{c+18} = c^2 \Rightarrow c^2 = 36$

$\Rightarrow\ c = \pm 6 \Rightarrow b = \dfrac{2+6}{2}, \dfrac{2-6}{2} = 4, -2.$

14. (6) : $3^{2n+2} - 8n - 9 = 9^{n+1} - 8(n+1) - 1$

$= (8+1)^{n+1} - 8(n+1) - 1$

$= 8^{n+1} + \binom{n+1}{1}8^n + \ldots + \binom{n+1}{n-1}8^2$, which is divisible by $8^2 = 2^6$.

15. (2) : $e^3 = \exp \lim\limits_{x\to 0}\left(\dfrac{x + \dfrac{f(x)}{x}}{x}\right)$

$$3 = \lim_{x\to 0} 1 + \frac{f(x)}{x^2} \Rightarrow \lim_{x\to 0}\frac{f(x)}{x^2} = 2$$

16. (0) : $x = 0$ in the identity gives

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = -12 + 12 = 0.$$

17. (3) : $I_n = \int_0^1 (x-1)^n e^x dx$

$$= (x-1)^n e^x\Big|_0^1 - n\int_0^1 (x-1)^{n-1}e^x\,dx$$

$I_n = (-1)^{n+1} - nI_{n-1} \Rightarrow I_0 = \int_0^1 e^x dx = e - 1$

$I_1 = 1 - I_0 = 1 - (e-1) = 2 - e$

$I_2 = -1 - 2I_1 = -1 - 2(2 - e) = 2e - 5$

$I_3 = 1 - 3I_2 = 1 - 3(2e - 5) = 16 - 6e$ $\therefore$ $n = 3$.

18. (2) : ω, ω^2 are the roots of the equation

$\dfrac{1}{a+x} + \dfrac{1}{b+x} + \dfrac{1}{c+x} = \dfrac{2}{x}$ ...(i)

$\Rightarrow\ x\,\Sigma\,(b+x)(c+x) = 2(a+x)(b+x)(c+x)$

$x^3 - (ab + bc + ca)x - 2abc = 0$

coeff. $x^2 = 0 \Rightarrow$ sum of the roots $= 0$

Also, $1 + \omega + \omega^2 = 0$

The third root is 1

(i) $\Rightarrow \dfrac{1}{a+1} + \dfrac{1}{b+1} + \dfrac{1}{c+1} = \dfrac{2}{1} = 2$.

PAPER-II

1. (d) : Let the plane be $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$

Volume $= k = \dfrac{abc}{6} \Rightarrow abc = 6k$ ....(i)

The foot of perpendicular from O on it is

$$\frac{x}{1/a} = \frac{y}{1/b} = \frac{z}{1/c} = \frac{1}{\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right)} = \alpha, \text{ say}$$

$\therefore$ $x^2 + y^2 + z^2 = \alpha$

$\Rightarrow$ $ax = by = cz = x^2 + y^2 + z^2$ ...(ii)

From (i), (ii) $\Rightarrow$ $6k = \dfrac{(x^2 + y^2 + z^2)^3}{xyz}$

or $(x^2 + y^2 + z^2)^3 = 6\,kxyz.$

2. (c): Let x be the length of an edge of the base and y be the height.

Given, $x^2 + 4xy = 400$ ...(i)

Volume, $V = x^2 y \Rightarrow V = \dfrac{x}{4}(400 - x^2) \Rightarrow \dfrac{dV}{dx} = 0$

$$\Rightarrow \frac{1}{4}(400 - x^2) + \frac{x}{4}(-2x) = 0$$

$$\Rightarrow \frac{-x^2}{4} + 100 - \frac{x^2}{2} = 0$$

$$\Rightarrow x = \frac{20}{\sqrt{3}},\ y = \frac{10}{\sqrt{3}}\ \therefore\ \frac{x}{y} = 2$$

3. (c): $S = \dfrac{1}{2}\sum\limits_{r=1}^{n-1}\left(1 + \cos\dfrac{2r\pi}{n}\right)$

$$= \frac{1}{2}\left[n - 1 + \sum_{r=1}^{n-1}\cos\frac{2r\pi}{n}\right]$$

$$T = \sum_{r=1}^{n-1}\cos\frac{2r\pi}{n}$$

$$2\sin\frac{\pi}{n}\cdot T = \sum_{r=1}^{n-1} 2\sin\frac{\pi}{n}\cos\frac{2r\pi}{n}$$

$$= \sum_{r=1}^{n-1}\left(\sin(2r+1)\frac{\pi}{n} - \sin(2r-1)\frac{\pi}{n}\right)$$

$$= \sin(2n-1)\frac{\pi}{n} - \sin\frac{\pi}{n} = -2\sin\frac{\pi}{n}$$

$$\therefore \quad T = -1,\ S = \frac{1}{2}(n-1-1) = \frac{n-2}{2}.$$

4. **(d) :** Let $a, ar, ar^2, \ldots\ldots$ be the G.P. $S = \dfrac{a(1-r^n)}{1-r}$

The reciprocals are in G. P. *i.e.* $\dfrac{1}{a}, \dfrac{1}{ar}, \dfrac{1}{ar^2}, \ldots\ldots$

$$R = \frac{\frac{1}{a}\left(1-\frac{1}{r^n}\right)}{1-\frac{1}{r}} = \frac{1}{a}\left(\frac{1-r^n}{1-r}\right)\frac{1}{r^{n-1}}$$

$$\therefore \quad \frac{S}{R} = a^2 r^{n-1} \qquad \ldots(1)$$

$$P = a \cdot ar \cdot ar^2 \ldots ar^{n-1} = a^n r^{(1+2+\ldots+(n-1))}$$
$$= a^n r^{(n-1)n/2}$$

$$\therefore \quad P^2 = a^{2n} \cdot r^{n(n-1)} = (a^2 r^{n-1})^n$$

$$= \left(\frac{S}{R}\right)^n \Rightarrow S = R \cdot P^{2/n} \qquad \text{(by (1))}$$

5. **(c):** $f'(x) = f(x) \Rightarrow f(x) = Ae^x$

$f(0) = 1 \Rightarrow f(x) = e^x,\ g(x) = x^2 - e^x$

$$I = \int_0^1 e^x(x^2 - e^x)dx = \int_0^1 (x^2e^x - e^{2x})dx$$

$$= (x^2 - 2x + 2)e^x - \frac{e^{2x}}{2}\Bigg|_0^1$$

$$= e - 2 - \frac{e^2}{2} + \frac{1}{2} = e - \frac{e^2}{2} - \frac{3}{2}.$$

6. **(b) :** $\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix} = 0 \Rightarrow k = \dfrac{33}{2}$

By cross multiplication rule using the last 2 equations,

$$\frac{x}{6-4k} = \frac{y}{8} = \frac{3}{9-2k}$$

$$\Rightarrow \quad \frac{x}{15} = \frac{y}{-2} = \frac{z}{6} \Rightarrow \frac{xy}{z^2} = \frac{-30}{36} = \frac{-5}{6}.$$

7. **(a, b, d) :** For $y \geq 0$, $x + y = 2y \Rightarrow y = x$

For $y < 0$, $x - y = 2y \Rightarrow y = \dfrac{x}{3}$

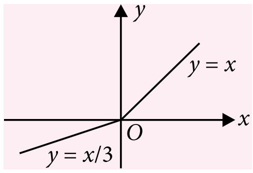
$f(x)$ is defined for all x.

$f(x)$ is continuous for all x.

$f(x)$ is differentiable for all x except $x = 0$.

$f'(x) = 1,\ x > 0,$

$f'(x) = \dfrac{1}{3},\ x < 0$



8. **(a, c):** $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = \cos\theta$

$$\Rightarrow \quad \vec{g} = \frac{\vec{a}+\vec{b}+\vec{c}}{3},\ |\vec{g}| = \frac{1}{3}\sqrt{|\vec{a}+\vec{b}+\vec{c}|^2}$$

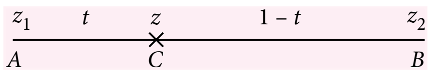
$$= \frac{1}{3}\sqrt{1+1+1+6\cos\theta} = \frac{1}{\sqrt{3}}\sqrt{1+2\cos\theta} \qquad \ldots\text{(i)}$$

$$\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2} \Rightarrow 0 \leq \cos\theta \leq \frac{1}{2},$$

$0 \leq 2\cos\theta \leq 1,\ 1 \leq 1 + 2\cos\theta \leq 2$

$$\therefore \quad |\vec{g}| \in \left[\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right] \text{ by (i).}$$

9. **(a, c, d) :** C divides AB in the ratio $t : 1 - t$



$\therefore \quad |z - z_1| = t\,|z_1 - z_2| \Rightarrow |z - z_2| = (1 - t)\,|z_1 - z_2|$

Adding, $|z - z_1| + |z - z_2| = |z_1 - z_2|$

AC and AB lie along the same line.

$\therefore \quad \arg(z - z_1) = \arg(z_2 - z_1) = \theta$, say

$z - z_1 = t(z_2 - z_1) = t|z_2 - z_1|\,e^{i\theta} = AB \cdot t \cdot e^{i\theta}$

$$\begin{vmatrix} z - z_1 & \bar{z} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = \begin{vmatrix} AB\cdot t\,e^{i\theta} & ABt\,e^{-i\theta} \\ AB\,e^{i\theta} & AB\,e^{-i\theta} \end{vmatrix}$$

$$= (AB)^2 t \begin{vmatrix} e^{i\theta} & e^{-i\theta} \\ e^{i\theta} & e^{-i\theta} \end{vmatrix} = 0.$$

10. **(a, c) :** The foci of the ellipse are

$$\left(\pm\sqrt{a^2 - b^2}, 0\right) = (\pm 3, 0)$$

The eccentricity of the ellipse is

$$\sqrt{1 - \frac{b^2}{a}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

The eccentricity of hyperbola is $\dfrac{5}{3}$

Let the hyperbola be $\dfrac{x^2}{p^2} - \dfrac{y^2}{q^2} = 1$

It passes through $(\pm 3, 0)$

$\therefore \quad p^2 = 9$

$$q^2 = p^2(e^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$$

$\therefore$ The hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

Foci are $\left(\pm\sqrt{p^2 + q^2}, 0\right) = (\pm 5, 0)$.

11. (a, d) : $S_n < \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \frac{1}{1+\frac{k}{n}+\left(\frac{k}{n}\right)^2}$

$$= \int_0^1 \frac{dx}{1+x+x^2} = \int_0^1 \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\Bigg|_0^1 = \frac{2}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}$$

$f(x)$ decreases in $(0, 1)$

$$\Rightarrow \frac{1}{n}\sum_{r=0}^{n-1} f\left(\frac{r}{n}\right) > \int_0^1 f(x)dx > \frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$$

$$\therefore T_n > \int_0^1 \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}.$$

12. (b, c, d) : The tangent is $y - 1 = (2 + b)(x - 1)$

Its intercepts on the x and y-axis are $\frac{1+b}{2+b}$ and $-(1 + b)$

$\frac{1}{2}\left|\frac{1+b}{2+b}\right||1+b| = 2 \quad \Rightarrow \quad b^2 + 2b + 1 = \pm 4(2 + b)$

$\Rightarrow (b + 3)^2 = 0,\ b^2 - 2b + 1 = 8 \Rightarrow b = -3, 1 \pm 2\sqrt{2}$

13. (b, d) : $x + y = 1$ ...(1)

$(c + 2)x + (c + 4)y = 6$ ...(2)

$(c + 2)^2 x + (c + 4)^2 y = 36$ ...(3)

$(2) - (c + 2)(1) \to (2)$, $(3) - (c + 2)^2(1) \to (3)$ gives

$x + y = 1$ ...(1)

$2y = 4 - c$ ...(2)

$4(c + 3)y = (c + 8)(4 - c)$ ...(3)

$(3) - 2(c + 3)(2) \to (3)$ gives

$0 = [(c + 8) - 2(c + 3)](4 - c)$

$\Rightarrow (4 - c)(2 - c) = 0 \Rightarrow c = 2, 4.$

14. (a, b, c) : Given $\frac{d}{dx}\left(y\frac{dy}{dx}\right) = x \Rightarrow y\frac{dy}{dx} = \frac{x^2}{2} + c$

$$\Rightarrow \frac{d}{dx}\left(\frac{y^2}{2}\right) = \frac{x^2}{2} + c$$

$\Rightarrow y^2 = \frac{x^3}{3} + \alpha x + \beta$ given $y(0) = 1, y(1) = 1$

$\Rightarrow \beta = 1, \alpha = \frac{-1}{3}$

$\therefore y^2 = f(x) = \frac{x^3 - x + 3}{3}, f'(x) = \frac{(3x^2 - 1)}{3} > 0$ for $x > 1$

$\therefore f(x)$ is monotonically increasing $\forall x \in (1, \infty)$

$$f\left(\frac{1}{\sqrt{3}}\right)f\left(-\frac{1}{\sqrt{3}}\right) > 0$$

$f(x) = 0$ has only one real root.

15. (d) : Clearly $S_n > C_n > T_n$ as 'x' is a proper fraction.

$\therefore x > x^2 > x^3$ and so on.

16. (c): We have $S_n = C_n + T_n$

$$\Rightarrow x\frac{((x^3)^{2n}-1)}{(x^3-1)} = x^2\frac{((x^3)^{2n}-1)}{(x^3-1)} + x^3\frac{((x^3)^{2n}-1)}{(x^3-1)}$$

But $x \neq 1$, as $x \in \left(0, \frac{\pi}{4}\right)$, so, we get $x = x^2 + x^3$

$\Rightarrow x^2 + x - 1 = 0\ (x \neq 0)$

$$\Rightarrow x = \frac{\sqrt{5}-1}{2} = 2\sin\frac{\pi}{10} \in \left(0, \frac{\pi}{4}\right)$$

17. (b) : Equation of hyperbola $(x - 1)(y - 2) = 4$

$\Rightarrow XY = 4$

If normal at $\left(ct, \frac{c}{t}\right)$ intersect curve again at $\left(ct', \frac{c}{t'}\right)$

then $t' = -\frac{1}{t^3}$

$2t = 4 \quad \Rightarrow \quad t = 2$

$(X', Y') \equiv \left(-\frac{1}{4}, -16\right)$

$\Rightarrow (\alpha, \beta - 29) \equiv \left(\frac{3}{4}, -14\right) \Rightarrow \alpha = \frac{3}{4}, \beta = 15$

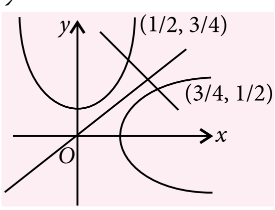
so $\frac{\beta}{\alpha} = \frac{15\times 4}{3} = 20$.

18. (a) : $2y\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

So, $d = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{1}{2\sqrt{2}}$

So $|\sin\alpha| = 1$

So number of solutions is 3.



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LOGICAL REASONING

Direction (1 - 2) : If 'P @ Q' means 'P is not greater than Q', 'P % Q' means 'P is not smaller than Q', 'P ★ Q' means 'P is neither smaller than nor equal to Q', 'P © Q' means 'P is neither greater than nor equal to Q and 'P $ Q' means 'P is neither greater than nor smaller than Q'.

In the following questions four statements showing relationship have been given, which are followed by four conclusions I, II, III and IV. Assuming that the given statements are true, find out which conclusion(s) is/are definitely true.

1. Statements : B % K, K $ T, T ★ F, H © F

Conclusions : I. B $ T

II. T © B

III. H © K

IV. F © B

(a) Only either I or II is true
(b) Only III is true
(c) Only III and IV are true
(d) Only either I or II and III and IV are true

2. Statements : W ★ B, B @ F, F © R, R $ M

Conclusions : I. W ★ F

II. M ★ B

III. R ★ B

IV. M ★ W

(a) Only I and IV are true
(b) Only II and III are true
(c) Only I and III are true
(d) Only II and IV are true

Direction (3 - 4) : B, M, K, H, T, R, D, W and A are sitting around a circle facing at the centre. R is third to the right of B. H is second to the right of A who is second to the right of R. K is third to the right of T, who is not an immediate neighbour of H. D is second to the left of T. M is fourth to the right of W. Now, answer the questions.

3. Who is third to the left of M?

(a) R (b) W (c) K (d) T

4. In which of the following combinations is the third person sitting in between the first and the second persons?

(a) WTR (b) BDT (c) MHD (d) WKR

5. In the given question three statements followed by three conclusions numbered I, II and III. You have to take the given statements to be true even if they seem to be at variance with commonly known of the given conclusions logically follows from the given statement, disregarding commonly known facts.

Statements : Some bags are plates. Some plates are chairs. All chairs are tables.

Conclusions : I. Some tables are plates.

II. Some chairs are bags.

III. No chair is bag.

(a) Only I follows
(b) Only either II or III follows
(c) Only I and either II or III follow
(d) None of these

6. In the given question statement followed by three assumptions numbered I, II and III. An assumption is something supposed or taken for granted. You have to consider the statement and the assumptions and decide which of the assumptions is implicit in the statement. Then decide which of the following options hold.

Statement : "Our school provides all facilities like school bus service, computer training, sports facilities. It also gives opportunity to participate in various extra-curricular activities apart from studies." An advertisement by a public school.

Assumptions: I. Nowadays extra-curricular activities assume more importance than studies.

II. Many parents would like to send their children to the school as it provides all the facilities.

III. Overall care of the child has become the need of the time as many women are working.

(a) Only I is implicit
(b) Only II is implicit
(c) Only I and II are implicit
(d) All I, II and III are implicit

7. Of the five villages P, Q, R, S and T situated close to each other, P is to west of Q, R is to the south of P, T is to the north of Q, and S is to the east of T. Then R is in which direction with respect to S?
(a) North-West (b) South-East
(c) South-West (d) Data Inadequate

8. A group of given digits is followed by four combinations of letters and symbols.
Digits are to be coded as per the scheme and conditions given below. You have to find out which of the four combinations correctly represents the group of digits. The serial number of that combination is you answer.

Digit :	5	1	4	8	9	3	6	2	7	0
Letter/Symbol :	Q	T	%	#	E	F	$	L	W	@

Conditions:
(i) If the first digit is odd and the last digit is even, their codes are to be swapped.
(ii) If the first as well as the last digit is even, both are to be coded by the code for first digit.
(iii) If the first digit is even and the last digit is odd, both are to be coded by the code for odd digit.

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(a) F#%$EF (b) F#%$EQ
(c) Q#%$EQ (d) None of these

9. If it is possible to make a meaningful word from the second, fourth, seventh and tenth letters of the word UNDENOMINATIONAL, using each letter only once, then the third letter of the word would be your answer. If more than one such word can be formed your answer would be 'X' whereas if no such word can be formed, your answer would be 'Z'.
(a) X (b) Z (c) M (d) N

10. Pointing to a man in the photograph a woman says, "He is the father of my daughter-in-law's brother-in-law". How is the man relate to the woman?
(a) husband (b) brother
(c) brother-in-law (d) father

11. A word and number arrangement machine when given an input line of words and numbers rearranges them following a particular rule in each step. The following is an illustration of input and steps of rearrangement.
Input : sky forward 17 over 95 23 come 40
Step I: come sky forward 17 over 95 23 40
Step II: come 95 sky forward 17 over 23 40
Step III: come 95 forward sky 17 over 23 40
Step IV: come 95 forward 40 sky 17 over 23
Step V: come 95 forward 40 over sky 17 23
Step VI: come 95 forward 40 over 23 sky
Step VI is the last step of the rearrangement of the above input.
As per the rules followed in the above steps. Which of the following will be step II for the given input?
(a) against 85 hire machine for 19 21 46
(b) against 85 machine 19 hire for 21 46
(c) against 85 machine hire for 19 21 46
(d) Cannot be determined

12. In the given question, two rows of numbers are given. The resultant number in each row is to be worked out separately based on the following rules and the question below the rows of numbers is to be answered. The operations of numbers progress from left to right.
Rules:
(i) If an odd number is followed by another composite odd number, they are to be multiplied.
(ii) If an even number number is followed by an odd number, they are to be added.
(iii) If an even number is followed by a number which is a perfect square, the even number to be subtracted from the perfect square.
(iv) If an odd number is followed by an even number, the second one is to be subtracted from the first one.
(v) If an odd number is followed by a prime odd number, the first number is to be divided by the second number.

27 18 3
21 x 9

If x is the resultant of the first row, what will be the resultant of the second row?

(a) 63 (b) 36 (c) 3 (d) 16

13. Find the missing number, if the given matrix follows a certain rule row-wise or column-wise.

7	4	5
8	7	6
3	3	?
29	19	31

(a) 3 (b) 4
(c) 5 (d) 6

14. Out of the car manufacturing companies A, B, C, D and E, the production of company B is more than that of company A but not more than that of company E. Production of company C is more than the production of company B but not as much as the production of company D. Considering the information to be true, which of the following is definitely true?

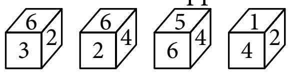
(a) Production of company D is highest of all the five companies.

(b) Company C produces more number of cars than Company E.

(c) The numbers of cars manufactured by companies E and C are equal

(d) Company A produces the lowest number of cars.

15. Four different positions of a dice have been shown below. What is the number opposite 1?



(a) 2 (b) 3 (c) 5 (d) 6

ANSWER KEY

1.	(d)	**2.**	(b)	**3.**	(a)	**4.**	(d)	**5.**	(c)
6.	(b)	**7.**	(c)	**8.**	(b)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(a)	**13.**	(c)	**14.**	(d)	**15.**	(d)

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PRACTICE PAPER 2017

Useful for All National/State Level Entrance Examinations

1. If $(1+\tan 1°)(1+\tan 2°)(1+\tan 3°)\ldots(1+\tan 45°) = 2^n$, then n is equal to

(a) 21 (b) 24 (c) 23 (d) 22

2. The value of 'a' for which the equation

$$4\operatorname{cosec}^2(\pi(a+x)) + a^2 - 4a = 0$$

has a real solution is

(a) $a = 1$ (b) $a = 2$

(c) $a = 3$ (d) none of these

3. If $x \in \left(\frac{3\pi}{2}, 2\pi\right)$, then value of the expression $\sin^{-1}(\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x)))$, is equal to

(a) $-\pi/2$ (b) $\pi/2$

(c) 0 (d) none of these

4. ABC is a triangle whose medians AD and BE are perpendicular to each other. If $AD = p$ and $BE = q$, then area of ΔABC is

(a) $\frac{2}{3}pq$ (b) $\frac{3}{2}pq$

(c) $\frac{4}{3}pq$ (d) $\frac{3}{4}pq$

5. The value of 'a' so that the sum of the squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is

(a) 2 (b) 0 (c) 3 (d) 1

6. The number of numbers lying between 100 and 500 that are divisible by 7 but not by 21 is

(a) 19 (b) 38 (c) 57 (d) 76

7. If a, b, c are in H.P., then straight line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$ always passes through a fixed point P with coordinates

(a) $(-1, -2)$ (b) $(-1, 2)$

(c) $(1, -2)$ (d) none of these

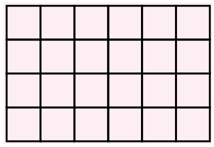
8. Integral part of $(5\sqrt{5}+11)^{2n+1}$ is

(a) even (b) odd

(c) cannot say anything

(d) neither even nor odd

9. Number of rectangles in figure shown which are not squares is



(a) 159 (b) 136

(c) 161 (d) none of these

10. Three equal circles each of radius r touch one another. The radius of the circle touching all the three given circles internally, is

(a) $(2+\sqrt{3})r$ (b) $\frac{(2+\sqrt{3})}{\sqrt{3}}r$

(c) $\frac{(2-\sqrt{3})}{\sqrt{3}}r$ (d) $(2-\sqrt{3})r$

11. The abscissae and ordinates of the end points A and B of the focal chord of the parabola $y^2 = 4x$ are respectively the roots of $x^2 - 3x + a = 0$ and $y^2 + 6y + b = 0$. The equation of the circle with AB as diameter is

(a) $x^2 + y^2 - 3x + 6y + 3 = 0$

(b) $x^2 + y^2 - 3x + 6y - 3 = 0$

(c) $x^2 + y^2 + 3x + 6y - 3 = 0$

(d) $x^2 + y^2 - 3x - 6y - 3 = 0$

12. A tangent having slope $-4/3$ to the ellipse $\frac{x^2}{18} + \frac{y^2}{32} = 1$ intersects the major and minor axes at A and B. If O is the origin, then the area of ΔOAB is

(a) 48 sq. units (b) 9 sq. units

(c) 24 sq. units (d) 16 sq. units

13. Let $f(x) = \frac{ax+b}{cx+d}$, then $f[f(x)] = x$ provided that

(a) $d = -a$ (b) $d = a$

(c) $a = b = 1$ (d) $a = b = c = d = 1$

14. If $f(x)$ is differentiable and strictly increasing function, then the value of $\lim_{x \to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is

(a) 1 (b) 0 (c) –1 (d) 2

15. Let $f: R \to R$ is a real valued function $\forall\ x, y \in R$ such that $|f(x) - f(y)| \le |x - y|^2$.
The function $h(x) = \int f(x)dx$ is
(a) everywhere continuous
(b) discontinuous at $x = 0$ only
(c) discontinuous at all integral points
(d) $h(0) = 0$

16. If $f(x) = \sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}}$, then $f'(x)$ at $x = 1.5$ is
(a) 0 (b) $-\sqrt{2}$
(c) $-\sqrt{3}$ (d) –4

17. The eccentricity of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ is changed at the rate of 0.1 m/sec. The time at which it will co-incident with the auxiliary circle is
(a) 2 seconds (b) 3 seconds
(c) 6 seconds (d) 5 seconds

18. The greatest of the numbers 1, $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $5^{1/5}$, $6^{1/6}$ and $7^{1/7}$ is
(a) $2^{1/2}$ (b) $3^{1/3}$
(c) $7^{1/7}$ (d) all are equal

19. If θ is the angle (semi-vertical) of a cone of maximum volume and given slant height, then $\tan\theta$ is given by
(a) 2 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$

20. If $\int_0^{\pi/2} \log\sin x\,dx = k$, then $\int_0^{\pi} \log(1+\cos x)dx$ is given by
(a) log2 + 4k (b) π log2 + 2k
(c) πlog2 + k (d) none of these

21. The area of the figure bounded by two branches of the curve $(y - x)^2 = x^3$ and the straight line $x = 1$ is
(a) 1/3 sq. units (b) 4/5 sq. units
(c) 5/4 sq. units (d) 3 sq. units

22. The differential equation $y\frac{dy}{dx} + x = k\ (k \in R)$ represents
(a) family of circles centered at y-axis
(b) family of circles centered at x-axis
(c) family of rectangular hyperbolas
(d) family of parabolas whose axis is x-axis

23. If $|z^2 - 1| = |z|^2 + 1$, then z lies on a
(a) circle (b) parabola
(c) ellipse (d) none of these

24. The probability that a particular day in the month of July is a rainy day is 3/4. Two persons whose credibility are $\frac{4}{5}$ and $\frac{2}{3}$ respectively, claim that 15th July was a rainy day. The probability that it was really a rainy day is _______.
(a) $\frac{3}{4}$ (b) $\frac{24}{25}$
(c) $\frac{8}{9}$ (d) none of these

25. In a quadrilateral $ABCD$,
let $\Delta = \begin{vmatrix} \cos A & \sin A & \cos(A+D) \\ \cos B & \sin B & \cos(B+D) \\ \cos C & \sin C & \cos(C+D) \end{vmatrix}$, then Δ is
(a) independent of A and B only
(b) independent of B and C only
(c) independent of A, B and C only
(d) independent of A, B, C and D all

26. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + ... + x^{16}$, then $f(A)$ is equal to
(a) 0 (b) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

27. A vector of magnitude 3, bisecting the angle between the lines in direction of the vectors $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$ and making an obtuse angle with $\vec{b}$ is
(a) $\frac{3\hat{i} - \hat{j}}{\sqrt{6}}$ (b) $\frac{\hat{i} + 3\hat{j} - 2\hat{k}}{\sqrt{14}}$
(c) $\frac{3(\hat{i} + 3\hat{j} - 2\hat{k})}{\sqrt{14}}$ (d) $\frac{3\hat{i} - \hat{j}}{\sqrt{10}}$

28. Equation of a plane through the line of intersection of planes $2x + 3y - 4z = 1$ and $3x - y + z + 2 = 0$ and parallel to $12x - y = 0$ is $(2x + 3y - 4z - 1) + \lambda(3x - y + z + 2) = 0$ then λ is
(a) –4 (b) 1/4 (c) 4 (d) –1/4

29. Negation of the proposition "If we control population growth, we prosper".

(a) If we do not control population growth, we prosper.
(b) It we control population growth, we do not prosper.
(c) We control population but we do not prosper.
(d) We do not control population, but we prosper.

30. The variance of the first n natural numbers is

(a) $\frac{n^2-1}{12}$ (b) $\frac{n^2-1}{6}$

(c) $\frac{n^2+1}{6}$ (d) $\frac{n^2+1}{12}$

SOLUTIONS

1. (c) : $(1 + \tan k°)(1 + \tan(45 - k)°) = 2$
$\Rightarrow$ n is equal to 23.

2. (b): We have, $4\operatorname{cosec}^2(\pi(a + x)) + a^2 - 4a = 0$

$\Rightarrow \operatorname{cosec}^2(\pi(a+x)) = \frac{4a-a^2}{4}$

$\Rightarrow \frac{4a-a^2}{4} \geq 1 \Rightarrow 4a - a^2 \geq 4$

$\Rightarrow a^2 - 4a + 4 \leq 0 \quad \Rightarrow \quad (a-2)^2 \leq 0$

$\therefore \quad a = 2$

3. (b): For $x \in \left(\frac{3\pi}{2}, 2\pi\right)$,

$\cos^{-1}(\cos x) = \cos^{-1}(\cos(2\pi - (2\pi - x)))$
$= \cos^{-1}(\cos(2\pi - x)) = 2\pi - x$

and $\sin^{-1}(\sin x) = \sin^{-1}(\sin(2\pi - (2\pi - x))) = x - 2\pi$

$\Rightarrow \cos^{-1}(\cos x) + \sin^{-1}(\sin x) = (2\pi - x) + (x - 2\pi) = 0.$

$\therefore \sin^{-1}(\cos(\cos^{-1}(\cos x) + \sin^{-1}(\sin x))) = \sin^{-1}(\cos(0))$
$= \sin^{-1}(1) = \pi/2$

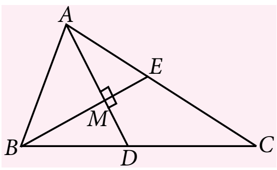
4. (a) : Since M is centroid, $AM : MD = 2 : 1$.
Also $BM : ME = 2 : 1$

$AM = \frac{2}{3}p,\ MD = \frac{1}{3}p,\ BM = \frac{2}{3}q$ and $ME = \frac{1}{3}q$

Area of triangle (ABM)

$= \frac{1}{2} \times AM \times BM$

$= \frac{1}{2} \times \frac{2}{3}p \times \frac{2}{3}q = \frac{2}{9}pq$



Area of ΔABC = 3area(ABM) = $\frac{2}{3}pq$

5. (d): $x^2 - (a - 2)x - a + 1 = 0$
Let α, β be the roots.
$\therefore \quad \alpha + \beta = (a - 2), \alpha\beta = 1 - a$
$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a - 2)^2 - 2(1 - a)$
$= a^2 - 4a + 4 - 2 + 2a = a^2 - 2a + 2 = (a - 1)^2 + 1$
It is minimum when $a = 1$

6. (b): The numbers which are divisible by 7 and lying between 100 and 500 are 105, 112, 119,...., 497.

$\therefore$ Total numbers $= \frac{497-105}{7} + 1 = 57$

The numbers which are divisible by 21 are
105, 126, 147, …., 483

$\therefore$ Total numbers $= \frac{483-105}{21} + 1 = 19$

So, required number = 57 – 19 = 38

7. (c) : We have, $2 \cdot \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0 \equiv \frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

$\Rightarrow x = 1, y = -2$

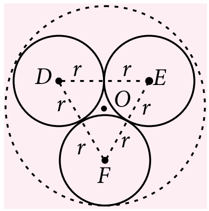
8. (a): $(5\sqrt{5}+11)^{2n+1} = I + f$ & let $(5\sqrt{5}-11)^{2n+1} = f'$

Also $I + f - f' = (5\sqrt{5}+11)^{2n+1} - (5\sqrt{5}-11)^{2n+1} = 2k$
i.e., Even integer

Hence $f - f' = 2k - I$ is an integer
But $-1 < f - f' < 1$, therefore $f - f' = 0$
$\Rightarrow I = 2k$ = even integer

9. (d)

10. (b): ΔDEF is equilateral with side $2r$. If radius of circumcircle of ΔDEF is R_1, then



Area of $\Delta DEF = \frac{\sqrt{3}}{4}(2r)^2 = \sqrt{3}r^2$

$\Rightarrow \sqrt{3}r^2 = \frac{2r \cdot 2r \cdot 2r}{4R_1} \quad \left[\because \Delta = \frac{abc}{4R}\right]$

$\Rightarrow R_1 = \frac{2r}{\sqrt{3}}$

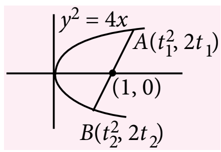
$\therefore$ Radius of the circle touching all the three given circles

$= r + R_1 = r + \frac{2r}{\sqrt{3}} = \frac{(2+\sqrt{3})r}{\sqrt{3}}$

11. (b): $t_1t_2 = -1$ as AB is focal chord
$x^2 - 3x + a = 0$; $x_1 + x_2 = 3$ and $x_1x_2 = a$
$y^2 + 6y + b = 0$; $y_1 + y_2 = -6$ and $y_1y_2 = b$

$x_1x_2 = \frac{1}{t_1^2} \cdot t_1^2 = 1 = a$

$y_1y_2 = 2t_1\left(-\frac{2}{t_1}\right) = -4 = b$



$a = 1, b = -4$
Equation of circle AB is diameter
$x^2 + y^2 - 3x + 6y - 3 = 0$

12. (c) : Any point on the ellipse $\frac{x^2}{(3\sqrt{2})^2}+\frac{y^2}{(4\sqrt{2})^2}=1$ can be taken as $(3\sqrt{2}\cos\theta, 4\sqrt{2}\sin\theta)$ and the slope of the tangent $=-\frac{b^2x}{a^2y}=-\frac{32(3\sqrt{2}\cos\theta)}{18(4\sqrt{2}\sin\theta)}=-\frac{4}{3}\cot\theta$ ...(1)

Given slope of the tangent $=-\frac{4}{3}$ ...(2)

From (1) and (2), we have

$\cot\theta = 1 \Rightarrow \theta = \pi/4$

Hence, the equation of the tangent is $\frac{x.\frac{1}{\sqrt{2}}}{3\sqrt{2}}+\frac{y.\frac{1}{\sqrt{2}}}{4\sqrt{2}}=1$

i.e. $\frac{x}{6}+\frac{y}{8}=1$

Hence, $A = (6, 0)$, $B = (0, 8)$

Area of $\Delta OAB = \frac{1}{2}\times 6\times 8 = 24$ sq. units

13. (a) : $fof(x)=\frac{a\left[\frac{ax+b}{cx+d}\right]+b}{c\left[\frac{ax+b}{cx+d}\right]+d}=x$

$\Rightarrow (ac + dc)x^2 + (bc + d^2 - bc - a^2)x - ab - bd = 0$

$\Rightarrow ac + dc = 0, a^2 - d^2 = 0, ab + bd = 0$

So $a = -d$

14. (c) : $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}\left(\frac{0}{0}\text{ form}\right)$

Apply L'Hospital rule, we get

$\lim_{x\to 0}\frac{f'(x^2)(2x)-f'(x)}{f'(x)}$

$f(x)$ is strictly increasing function. So, $f'(x) > 0$

$\Rightarrow \frac{f'(0)(2\times 0)-f'(0)}{f'(0)}=-\frac{f'(0)}{f'(0)}=-1$

15. (a) : Since, $|f(x) - f(y)| \le |x - y|^2, x \ne y$

$\therefore \left|\frac{f(x)-f(y)}{x-y}\right|\le|x-y|$

Taking lim as $y \to x$, we get

$\lim_{y\to x}\left|\frac{f(x)-f(y)}{x-y}\right|\le\lim_{y\to x}|x-y|$

$\Rightarrow \left|\lim_{y\to x}\frac{f(x)-f(y)}{x-y}\right|\le\left|\lim_{y\to x}(x-y)\right|$

$\Rightarrow |f'(x)|\le 0 \Rightarrow |f'(x)|=0 \quad (\because |f'(x)\ge 0|)$

$\therefore f'(x) = 0 \Rightarrow f(x) = c$ (constant)

$\therefore h(x)=\int f(x)dx=\int c\,dx = cx+d$

where d is constant of integration.

$\therefore$ $h(x)$ of a linear function of x which is continuous for all $x\in R$.

16. (b): $f(x)=\sqrt{(x-1)+4-4\sqrt{x-1}}+\sqrt{(x-1)+9-6\sqrt{x-1}}$

$=\left|\sqrt{x-1}-2\right|+\left|\sqrt{x-1}-3\right|$

$\Rightarrow f(x)=(2-\sqrt{x-1})+(3-\sqrt{x-1})=5-2\sqrt{x-1}$

$f'(x)=\frac{-2}{2\sqrt{x-1}}$

$\therefore f'(1.5)=\frac{-1}{\sqrt{\frac{3}{2}-1}}=-\sqrt{2}$

17. (d): $\frac{de}{dt}=-0.1$ m/sec

$\Rightarrow e_t = -0.1\,t + e_0$

We have, $3 = 4(1 - e_0^2)$

$\Rightarrow \frac{3}{4}=1-e_0^2 \Rightarrow e_0=\frac{1}{2}$

$\therefore e_t=-0.1t+\frac{1}{2}$, given $e_t=0$

$\Rightarrow 0.1t=\frac{1}{2}$ or $t=5$ seconds

18. (b): Assuming the function

$f(x) = x^{1/x}\ \{x > 0\}$

$\therefore f'(x)=x^{1/x}\left[\frac{d}{dx}\left(\frac{1}{x}\log x\right)\right]$

$\Rightarrow f'(x)=x^{1/x}\left[-\frac{\log x}{x^2}+\frac{1}{x^2}\right]$

$\Rightarrow f'(x)=\frac{x^{1/x}}{x^2}(1-\log x)$

Clearly when $x > e$, $f'(x)$ will be negative $f(x)$ is decreasing.

When $0 < x < e$, $f'(x)$ will be positive $f(x)$ is increasing.

It means $3^{1/3} > 4^{1/4} > 5^{1/5} > 6^{1/6} > 7^{1/7}$

Now compare (1) and $(2)^{1/2}$ and $(3)^{1/3}$

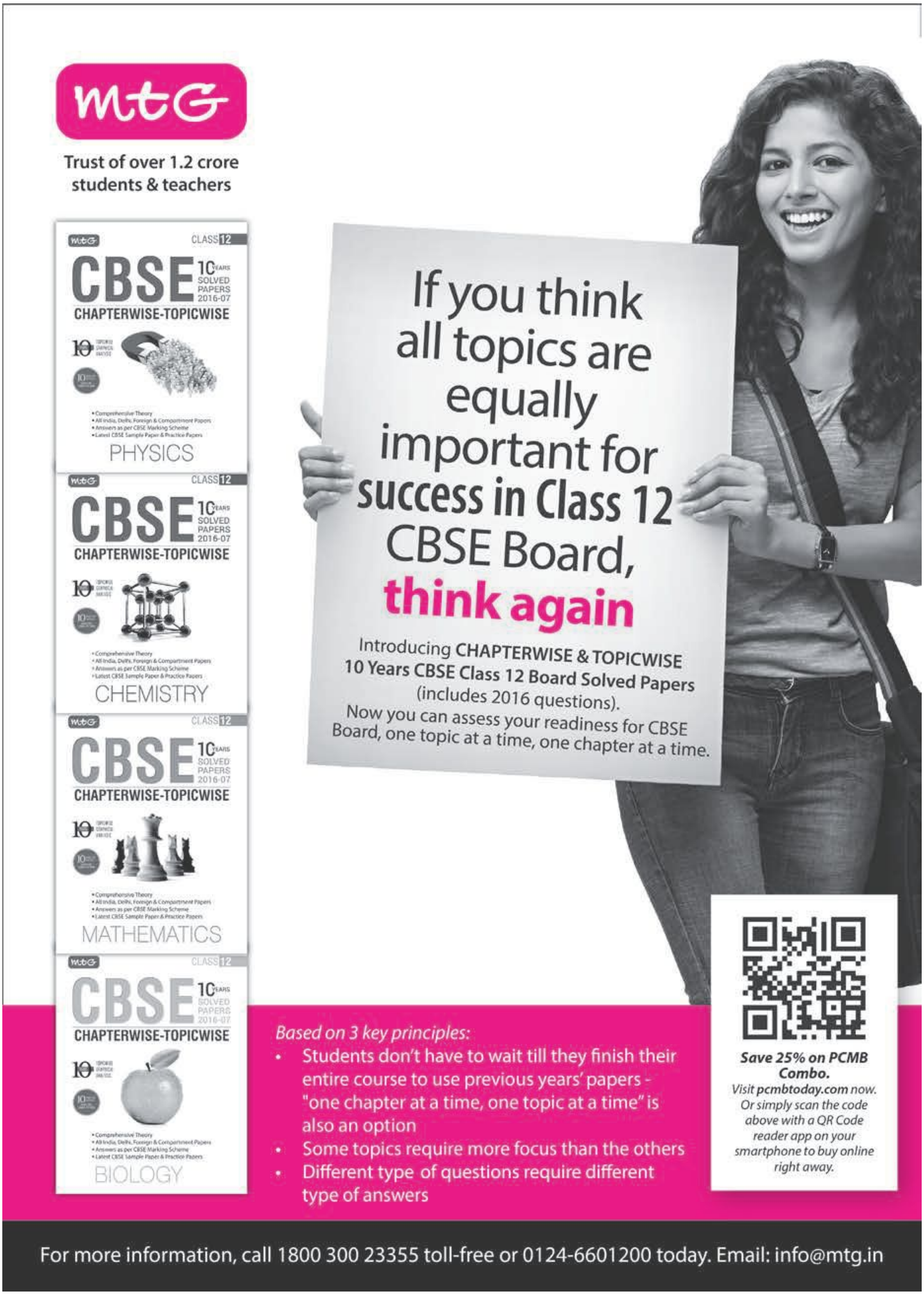
Apply same power on given three numbers.

$(1)^6$	$(2)^{1/2\times 6}$	$(3)^{1/3\times 6}$
1	2^3	3^2
1 <	8 <	9
(1) <	$(2)^{1/2}$ <	$(3)^{1/3}$

So maximum number is $(3)^{1/3}$.

19. (c) : Let $OB = l$, $OA = l\cos\theta$ and $AB = l\sin\theta$ $(0 \le \theta \le \pi/2)$. Then

$V=\frac{\pi}{3}(AB)^2(OA)=\frac{\pi}{3}l^3\sin^2\theta\cos\theta$



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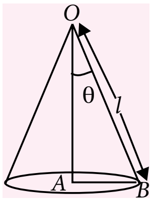
$\Rightarrow \quad \dfrac{dV}{d\theta} = \dfrac{\pi}{3} l^3 \sin\theta(3\cos^2\theta - 1)$



For maximum,

$\dfrac{dV}{d\theta} = 0 \quad \Rightarrow \quad \theta = 0$ or $\cos\theta = \dfrac{1}{\sqrt{3}}$.

Also $V(0) = 0$, $V(\pi/2) = 0$

and $V\left(\cos^{-1}\dfrac{1}{\sqrt{3}}\right) = \dfrac{2\pi l^3}{9\sqrt{3}}$

Hence V is maximum when $\cos\theta = \dfrac{1}{\sqrt{3}}$.

i.e. $\tan\theta = \sqrt{2}$

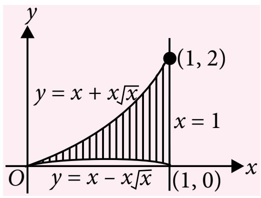
20. (d): $I = \int_0^{\pi} \log(1+\cos x)\,dx = \int_0^{\pi} \log\left(2\cos^2\dfrac{x}{2}\right)dx$

$= \int_0^{\pi} \log 2 dx + \int_0^{\pi} \log\cos^2\dfrac{x}{2}dx = \pi\log 2 + 2\int_0^{\pi}\log\cos\dfrac{x}{2}dx$

21. (b): Given curve is $(y - x)^2 = x^3$

$\Rightarrow \quad y - x = \pm x\sqrt{x}$

$\Rightarrow \quad y = x \pm x\sqrt{x}$



$\therefore$ Required area

$= \left|\int_0^1 [(x + x\sqrt{x} - (x - x\sqrt{x})]dx\right|$

$= \left|2\int_0^1 x^{3/2}dx\right| = \left|2.\dfrac{2}{5}\right| = \dfrac{4}{5}$ sq. units

22. (b): We have, $y\dfrac{dy}{dx} = (k - x) \Rightarrow y\,dy = (k - x)dx$

Integrate both sides, we get

$\int y\,dy = \int (k-x)\,dx \quad \Rightarrow \quad \dfrac{y^2}{2} = -\dfrac{(k-x)^2}{2} + c$

$\Rightarrow \quad \dfrac{(x-k)^2}{2} + \dfrac{y^2}{2} = c$, represents family of circles whose centre is at $(k, 0)$.

23. (d): On putting $z = x + iy$, the equation is same as

$|x^2 - y^2 + 2ixy - 1| = x^2 + y^2 + 1$

$\Rightarrow (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$

$\Rightarrow x = 0$

$\Rightarrow$ z lies on imaginary axis, so (a), (b), (c) are ruled out.

24. (b): A : Event that first man speaks truth

B : Event that second man speaks truth

R : Day is rainy

$P(R) = \dfrac{P(A\cap B)\cdot P(R)}{P(A\cap B)\cdot P(R) + P(A'\cap B')\cdot P(R')}$

$= \dfrac{\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{3}{4}}{\frac{4}{5}\cdot\frac{2}{3}\cdot\frac{3}{4} + \frac{1}{5}\cdot\frac{1}{3}\cdot\frac{1}{4}} = \dfrac{24}{25}$

25. (d): $C_3 \to C_3 - C_1\cos D + C_2\sin D = 0$

So $\Delta = 0$, hence Δ is independent of A, B, C, D all.

26. (b): $f(A) = I + A + A^2 + \ldots + A^{16}$

$A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Similarly $A^4 = A^5 = \ldots\ldots = A^{16} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$f(A) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + \ldots + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

27. (c) : A vector bisecting the angle between $\vec{a}$ and $\vec{b}$ is $\dfrac{\vec{a}}{|\vec{a}|} \pm \dfrac{\vec{b}}{|\vec{b}|}$.

In this case, $\dfrac{2\hat{i}+\hat{j}-\hat{k}}{\sqrt{6}} \pm \dfrac{\hat{i}-2\hat{j}+\hat{k}}{\sqrt{6}}$

i.e. $\dfrac{3\hat{i}-\hat{j}}{\sqrt{6}}$ or $\dfrac{\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{6}}$

A vector of magnitude 3 along these vectors is

$\dfrac{3(3\hat{i}-\hat{j})}{\sqrt{10}}$ or $\dfrac{3(\hat{i}+3\hat{j}-2\hat{k})}{\sqrt{14}}$

Now, $\dfrac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k})$ is negative and hence $\dfrac{3}{\sqrt{14}}(\hat{i}+3\hat{j}-2\hat{k})$ makes an obtuse angle with $\vec{b}$.

28. (c) : $(2 + 3\lambda)x + (3 - \lambda)y + (-4 + \lambda)z = 1 - 2\lambda$

It is parallel to $12x - y = 0$ if

$\dfrac{2+3\lambda}{12} = \dfrac{3-\lambda}{-1} = \dfrac{\lambda-4}{0} \quad \Rightarrow \lambda = 4$

29. (c) : p : we control population, q : we prosper.

$\therefore$ we have $p \Rightarrow q$

Its negation is $\sim(p \Rightarrow q)$ *i.e.* $p \wedge \sim q$

i.e. we control population but we do not prosper

30. (a) : $\text{Variance} = (\text{S.D.})^2 = \dfrac{1}{n}\Sigma x^2 - \left(\dfrac{\Sigma x}{n}\right)^2; \left(\because \bar{x} = \dfrac{\Sigma x}{n}\right)$

$= \dfrac{n(n+1)(2n+1)}{6n} - \left(\dfrac{n(n+1)}{2n}\right)^2 = \dfrac{n^2-1}{12}$

PRACTICE PAPER
JEE ADVANCED

Exam on 21st May 2017

*ALOK KUMAR, B.Tech, IIT Kanpur

ONLY ONE OPTION CORRECT TYPE

1. The number of ways in which the squares of a 8×8 chess board can be painted red or blue so that each 2×2 square has two red and two blue squares is
(a) 2^9 (b) $2^9 - 1$
(c) $2^9 - 2$ (d) none of these

2. Box A contains black balls and box B contains white balls take a certain number of balls from A and place them in B, then take same number of balls from B and place them in A. The probability that number of white balls in A is equal to number of black balls in B is equal to
(a) 1/2 (b) 1/3
(c) 2/3 (d) none of these

3. The number of planes that are equidistant from four non-coplanar points is
(a) 3 (b) 4 (c) 7 (d) 9

4. 5 different games are to be distributed among 4 children randomly. The probability that each child get atleast one game is
(a) $\frac{1}{4}$ (b) $\frac{15}{64}$
(c) $\frac{21}{64}$ (d) none of these

5. Let a, b, c be distinct non-negative numbers and the vectors $a\hat{i}+a\hat{j}+c\hat{k}$, $\hat{i}+\hat{k}$, $c\hat{i}+c\hat{j}+b\hat{k}$ lie in a plane, then the quadratic equation $ax^2 + 2cx + b = 0$ has
(a) real and equal roots
(b) real and unequal roots
(c) both roots real and positive
(d) none of these

6. If three equations are consistent $(a + 1)^3x + (a + 2)^3y = (a + 3)^3$; $(a + 1)x + (a + 2)y = a + 3$, $x + y = 1$, then $a =$
(a) 1 (b) 2 (c) -2 (d) 3

7. If three one by one squares are selected at random from the chessboard, then the probability that they form the letter 'L' is
(a) $\frac{196}{{}^{64}C_3}$ (b) $\frac{49}{{}^{64}C_3}$ (c) $\frac{36}{{}^{64}C_3}$ (d) $\frac{98}{{}^{64}C_3}$

8. $\lim\limits_{x\to\infty} \int_0^{x^2/2} \frac{t^2}{x^2(1+t^2)}dt$ is equal to
(a) 1/4 (b) 1/2
(c) 1 (d) none of these

9. If $a, b, c > 0$, then the minimum value of $(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ must be
(a) 6 (b) 8
(c) 3 (d) none of these

10. The value of $\int \frac{(x^2+1)}{(x^4-x^2+1)\cot^{-1}\left(x-\frac{1}{x}\right)}dx$ will be
(a) $\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right|-c$
(b) $-\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right|+c$
(c) $\ln\left|\cot^{-1}\left(x+\frac{1}{x}\right)\right|-c$
(d) none of these

* Alok Kumar is a winner of INDIAN NATIONAL MATHEMATICS OLYMPIAD (INMO-91).
He trains IIT and Olympiad aspirants.

11. The highest power of 3 contained in (70)! must be equal to
(a) 16 (b) 24
(c) 32 (d) 48

12. If A is a nilpotent matrix of index 2, then for any positive integer n, $A(I + A)^n$ is equal to
(a) A^{-1} (b) A (c) A^n (d) I_n

13. If $\vec{r}_a, \vec{r}_b, \vec{r}_c$ are unit vectors such that $\vec{r}_a \cdot \vec{r}_b = 0 = \vec{r}_a \cdot \vec{r}_c$ and the angle between $\vec{r}_b$ and $\vec{r}_c$ is $\pi/3$. Then the value of $|\vec{r}_a \times \vec{r}_b - \vec{r}_a \times \vec{r}_c|$ is
(a) 1/2 (b) 1
(c) 2 (d) none of these

14. The curve $xy = c(c > 0)$ and the circle $x^2 + y^2 = 1$ touch at two points, then distance between the points of contact is
(a) 1 (b) 2
(c) $2\sqrt{2}$ (d) none of these

15. A variable circle is drawn to pass through (1, 0) and to touch the line $x + y = 0$. Let $S = 0$ represent the locus of the centre of the circle. Then $S = 0$ represents
(a) pair of parallel lines
(b) circle
(c) parabola
(d) none of these

16. If α, β and γ are the roots of the equation $x^2(px + q) = r(x + 1)$. Then the value of determinant

$$\begin{vmatrix} 1+\alpha & 1 & 1 \\ 1 & 1+\beta & 1 \\ 1 & 1 & 1+\gamma \end{vmatrix} \text{ is}$$

(a) $\alpha\beta\gamma$ (b) $1+\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}$
(c) 0 (d) none of these

17. If $I_{m,n} = \int \cos^m x \cos nx\, dx$, then the value of $(m + n)I_{m,n} - mI_{m-1,n-1}$ $(m, n \in N)$ is equal to
(a) $\dfrac{\cos^m x \cos nx}{n}+c$ (b) $\cos^m x \sin nx + c$
(c) $\dfrac{\cos^{m-1} x \cos nx}{n}+c$ (d) $-\cos^m x \cos nx + c$

18. If $I = \int_0^{\pi/2} \cos 2nx \log \cos x dx$. Then $I =$
(a) $\int_0^{\pi/2} \cot x \cos 2nx dx$ (b) $\int_0^{\pi/2} \cot x \sin 2nx dx$
(c) $\int_0^{\pi/2} \tan x \cos 2nx dx$ (d) $\dfrac{1}{2n}\int_0^{\pi/2} \tan x \sin 2nx dx$

19. If $y = e^{\{(\cos^2 x+\cos^4 x+\ldots\infty)\log_e 2\}}$ satisfies the equation $x^2 - 9x + 8 = 0$, then find the value of $\dfrac{\sin x}{\sin x+\cos x}, 0 < x < \dfrac{\pi}{2}$.
(a) $\dfrac{\sqrt{3}+1}{2}$ (b) $\dfrac{1}{\sqrt{3}-1}$
(c) $\dfrac{1}{\sqrt{3}+1}$ (d) none of these

20. If p is a prime number $(p \neq 2)$, then the difference $[(2+\sqrt{5})^p] - 2^{p+1}$ is always divisible by (where [.] denotes the greatest integer function)
(a) $p + 1$ (b) p
(c) $2p$ (d) $5p$

21. Let $a, b, c \in R$ such that no two of them are equal and satisfy $\begin{vmatrix} 2a & b & c \\ b & c & 2a \\ c & 2a & b \end{vmatrix} = 0$, then equation $24ax^2 + 4bx + c = 0$ has
(a) atleast one root in [0, 1/2]
(b) atleast one root in [–1/2, 0)
(c) atleast one root in [1, 2]
(d) atleast two roots in [0, 2]

22. $I = \int \dfrac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}} d\theta$ is equal to
(a) $\tan\dfrac{\theta}{2}+c$ (b) $\dfrac{7}{11}\left[\tan\left(\dfrac{\theta}{2}\right)^{11/7}\right]+c$
(c) $7\left(\tan\dfrac{\theta}{2}\right)^7+c$ (d) none of these

23. Number of solutions of the equation $1 + e^{|x|} = |x|$ is/are
(a) 0 (b) 1 (c) 2 (d) 4

24. The value of $\lim\limits_{n\to\infty} \sum_{r=1}^{n}\left(\sum_{t=0}^{r-1} \dfrac{1}{5^n} \cdot {}^nC_r \cdot {}^rC_t \cdot 3^t\right)$ is equal to
(a) 4 (b) 3
(c) 1 (d) none of these

ONE OR MORE THAN ONE OPTION CORRECT TYPE

25. The values of a for which the integral $\int_0^2 |x-a|\,dx \ge 1$ is satisfied are

(a) $[2, \infty)$ (b) $(-\infty, 0]$
(c) $(0, 2)$ (d) none of these

26. The number of ways of choosing triplet (x, y, z) such that $z \ge \max\{x, y\}$ and $x, y, z \in (1, 2, ...n, n+1)$ is

(a) ${}^{n+1}C_3 + {}^{n+2}C_3$ (b) $\frac{1}{6}n(n+1)(2n+1)$
(c) $1^2 + 2^2 + ... + n^2$ (d) $2({}^{n+2}C_3) - {}^{n+1}C_2$

27. If inside a big circle exactly 24 small circles, each of radius 2, can be drawn in such a way that each small circle touches the big circle and also touches both its adjacent small circles, then radius of the big circle is

(a) $2\left(1+\operatorname{cosec}\frac{\pi}{4}\right)$ (b) $\left(\frac{1+\tan\frac{\pi}{24}}{\cos\frac{\pi}{24}}\right)$

(c) $2\left(1+\operatorname{cosec}\frac{\pi}{12}\right)$ (d) $\frac{2\left(\sin\frac{\pi}{48}+\cos\frac{\pi}{48}\right)^2}{\sin\frac{\pi}{24}}$

28. Consider $f(x) = (\cos\theta + x\sin\theta)^n - \cos n\theta - x\sin n\theta$, $(n \in N)$ then $f(x)$ is always divisible by

(a) $x + i$ (b) $x - i$
(c) $x^2 + 1$ (d) none of these

29. If a, b, c are the sides of a triangle, then

$\frac{a}{c+a-b}+\frac{b}{a+b-c}+\frac{c}{b+c-a}$ can take value(s)

(a) 1 (b) 2 (c) 3 (d) 4

30. The triangle formed by the normal to the curve $f(x) = x^2 - ax + 2a$ at the point (2, 4) and the coordinate axes lies in second quadrant. If its area is 2 sq. units, then a can be

(a) 2 (b) 17/4 (c) 5 (d) 19/4

31. If $\Delta(x)=\begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix}$

$= ax^3 + bx^2 + cx + d$, then

(a) $a = 3$ (b) $b = 0$
(c) $c = 0$ (d) none of these

32. If $f(x)=\int_1^x \frac{\ln t}{t(1+t)}dt$, then

(a) $f\left(\frac{1}{x}\right)=-\int_1^x \frac{\ln t}{t(1+t)}dt$

(b) $f\left(\frac{1}{x}\right)=\int_1^x \frac{\ln t}{t^2(1+t)}dt$

(c) $f(x)+f\left(\frac{1}{x}\right)=0$

(d) $f(x)+f\left(\frac{1}{x}\right)=\frac{1}{2}(\ln x)^2$

33. A, B, C are three events for which $P(A) = 0.4$, $P(B) = 0.6$, $P(C) = 0.5$, $P(A \cup B) = 0.75$, $P(A \cap C) = 0.35$ and $P(A \cap B \cap C) = 0.2$. If $P(A \cup B \cap C) \ge 0.75$, then $P(B \cap C)$ can take values

(a) 0.1 (b) 0.2
(c) 0.3 (d) 0.5

COMPREHENSION TYPE

Paragraph for Q. No. 34 to 36

Let n be a positive integer such that $I_n = \int x^n\sqrt{a^2-x^2}\,dx$.

34. The value of I_1 is

(a) $\frac{2}{3}(a^2-x^2)^{1/2}+C$

(b) $\frac{1}{3}(a^2-x^2)^{3/2}+C$

(c) $-\frac{2}{3}(a^2-x^2)^{3/2}+C$

(d) $-\frac{1}{3}(a^2-x^2)^{3/2}+C$

35. The value of the expression $\frac{\int_0^a x^4\sqrt{a^2-x^2}\,dx}{\int_0^a x^2\sqrt{a^2-x^2}\,dx}$ is equal to

(a) $\frac{a^2}{6}$ (b) $\frac{3a^2}{2}$

(c) $\frac{3a^2}{4}$ (d) $\frac{a^2}{2}$

36. If $I_n = \frac{-x^{n-1}(a^2-x^2)^{3/2}}{n+2} + kI_{n-2}$, then the values k is

(a) $\frac{n-1}{n+2}$ (b) $\frac{n+2}{n-1}$

(c) $\left(\frac{n-1}{n+2}\right)a^2$ (d) $\left(\frac{n+2}{n-1}\right)a^2$

MATRIX MATCH TYPE

37. Match the following .

	List-I	List-II	
A.	Normal of parabola $y^2 = 4x$ at P and Q meets at R $(x_2, 0)$ and tangents at P and Q meets at $T(x_1, 0)$, then if $x_2 = 3$, then the area of quadrilateral $PTQR$ is	(i)	3
B.	The length of latus rectum plus tangent PT will be	(ii)	6
C.	The quadrilateral $PTQR$ can be inscribed in a circle, then the value of $\frac{\text{circumference}}{4\pi}$ will be	(iii)	1
D.	The number of normals that can be drawn to the parabola from $R(x_2, 0)$	(iv)	8

38. Match the following.

	List-I	List-II	
A.	Area bounded by the parabola $y^2 = 4x$ and the line $y = x$ is	(i)	1/2
B.	$\int_0^{\pi}(\sin^5 x+\cos^5 x)dx$ is equal to	(ii)	–11
C.	If $S = 0$ be the equation of the hyperbola $x^2 + 4xy + 3y^2 - 4x + 2y + 1 = 0$ then the value of k for which $S + k = 0$ represents its asymptotes is	(iii)	8/3
D.	The value of α for which $\alpha\hat{i}+2\hat{j}+\hat{k}$, lies in the plane containing three points $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+2\hat{j}+\hat{k}$ and $3\hat{i}-\hat{k}$ is	(iv)	$\frac{16}{15}$
		(v)	–22

INTEGER ANSWER TYPE

39. The number of real solutions of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is ______.

40. The figures 4, 5, 6, 7, 8 are written in every possible order. The sum of the number of numbers greater than 56000 is ______.

SOLUTIONS

1. **(c) :** Number of ways when the squares are alternating colour in first column is 2^8.
Number of ways when the squares in the first column are not alternating colour = $2^8 - 2$

$\therefore$ Required number of ways = $2^9 - 2$

2. **(d)**

3. **(c) :** Let the points be A, B, C and D.
The number of planes which have three points on one side and the fourth point on the other side = 4. The number of planes which have two points on each side of the plane = 3
Hence, number of planes = 7.

4. **(b) :** Total ways of distribution = 4^5
Total ways of distribution so that each child get atleast one game $= 4^5 - {}^4C_1 3^5 + {}^4C_2 2^5 - {}^4C_3 = 240$

$\therefore$ Required probability $= \frac{240}{4^5} = \frac{15}{64}$

5. **(a) :** $\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$

$\Rightarrow$ $c^2 - ab = 0$
and from $ax^2 + 2cx + b = 0$, we have
$D = 4c^2 - 4ab = 4(c^2 - ab) = 0$
So, roots are real and equal.

6. **(c) :** Since the equations are consistant.
$\therefore$ $\Delta = 0$

$$\Rightarrow \begin{vmatrix} (a+1)^3 & (a+2)^3 & -(a+3)^3 \\ (a+1) & (a+2) & -(a+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

Put $u = a + 1$, $v = a + 2$ and $w = a + 3$
$\Rightarrow$ $u - v = -1$, $v - w = -1$, $w - u = 2$
Also, $u + v + w = 3a + 6$

$$\therefore \begin{vmatrix} u^3 & v^3 & -w^3 \\ u & v & -w \\ 1 & 1 & -1 \end{vmatrix} = 0 \Rightarrow (u-v)(v-w)(w-u)(u+v+w) = 0$$

$\Rightarrow$ $(-1)(-1)(2)(3a + 6) = 0$ $\Rightarrow a = -2$

7. (a) : Total number of outcomes = ${}^{64}C_3$.
Let 'E' be the event of selecting 3 squares which form the letter 'L'.
No. of ways selecting squares consisting of 4 unit squares = ${}^7C_1 \times {}^7C_1 = 49$
Each square with 4 unit squares forms 4L-shapes consisting of 3 unit squares.

$$\therefore \quad n(E) = 7\times 7\times 4 = 196, \quad \therefore P(E) = \frac{196}{{}^{64}C_3}$$

8. (b) : $\lim\limits_{x\to\infty} \int_0^{x^2/2} \frac{t^2}{x^2(1+t^2)}dt$

$$= \lim_{x\to\infty} \frac{\int_0^{x^2/2} \frac{t^2}{(1+t^2)}dt}{x^2}\left(\frac{\infty}{\infty}\text{form}\right) = \lim_{x\to\infty}\left[\frac{\frac{\left(\frac{x^2}{2}\right)^2}{1+\left(\frac{x^2}{2}\right)^2} d\left(\frac{x^2}{2}\right)}{2x}\right]$$

$$= \lim_{x\to\infty} \frac{\frac{x^4}{4+x^4}x}{2x} = \lim_{x\to\infty}\frac{x^4}{2x^4+8} = \frac{1}{2}$$

9. (d)

10. (b) : Let $I = \int \frac{(x^2+1)}{(x^4-x^2+1)\cot^{-1}\left(x-\frac{1}{x}\right)}dx$

$$\Rightarrow \quad I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}-2+1\right)\cot^{-1}\left(x-\frac{1}{x}\right)}dx$$

Put $x - \frac{1}{x} = t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt$

Let $\cot^{-1}t = u$

$$\therefore \quad I = \int \frac{dt}{(t^2+1)\cot^{-1}t} \Rightarrow \quad -\frac{1}{(1+t^2)}dt = du$$

$$\therefore \quad I = \int \frac{-du}{u}$$

$$= -\ln|u| + c = -\ln\left|\cot^{-1}\left(x-\frac{1}{x}\right)\right| + c$$

11. (c) : $\left[\frac{70}{3}\right]+\left[\frac{70}{9}\right]+\left[\frac{70}{27}\right] = 23+7+2 = 32$

12. (b) : $A^2 = 0 \cdot A^3 = A^4 = ... = A^n = 0$
then $A(I \pm A)^n = A(I \pm nA) = A \pm nA^2 = A$

13. (b) : $\vec{r}_a \cdot \vec{r}_b = 0 \;\Rightarrow\; \vec{r}_a \perp \vec{r}_b$,

$\vec{r}_a \cdot \vec{r}_c = 0 \;\Rightarrow\; \vec{r}_a \perp \vec{r}_c$

$\Rightarrow\; \vec{r}_a \perp \vec{r}_b - \vec{r}_c$

$$\therefore \quad |\vec{r}_a \times (\vec{r}_b - \vec{r}_c)| = |\vec{r}_a \times \vec{r}_b - \vec{r}_a \times \vec{r}_c|$$

$$= |\vec{r}_a||\vec{r}_b - \vec{r}_c| = |\vec{r}_b - \vec{r}_c|$$

Now,

$$|\vec{r}_b - \vec{r}_c|^2 = |\vec{r}_b|^2 + |\vec{r}_c|^2 - 2|\vec{r}_b||\vec{r}_c|\cos\frac{\pi}{3} = 2 - 2\times\frac{1}{2} = 1$$

$$\Rightarrow \quad |\vec{r}_b - \vec{r}_c| = 1$$

14. (b) : The curve $xy = c$ and the circle $x^2 + y^2 = 1$ touches each other, so

$x^2 + \frac{c^2}{x^2} - 1 = 0 \Rightarrow x^4 - x^2 + c^2 = 0$ will have equal roots

so, $(-1)^2 - 4c^2 = 0$

$$\Rightarrow \quad 4c^2 = 1 \;\Rightarrow\; c^2 = \frac{1}{4} \;\Rightarrow\; c = \pm\frac{1}{2}$$

$\therefore$ Roots of the equation $x^4 - x^2 + \frac{1}{4} = 0$ are

$$x = \pm\frac{1}{\sqrt{2}} \;\Rightarrow\; y = \pm\frac{1}{\sqrt{2}}$$

Hence, distance between the points of contact = 2 units.

15. (c) : Let $P(h, k)$ be the centre of the circle. Since the circle passes through (1, 0).

$\therefore$ Its radius $= \sqrt{(h-1)^2 + k^2}$

It also touches the line $x + y = 0$,

so $\frac{h+k}{\sqrt{2}} = \pm\sqrt{(h-1)^2 + k^2}$

Hence locus of the centre $P(h, k)$ is

$S \equiv x^2 + y^2 - 2xy - 4x + 2 = 0$

Here, $h^2 = ab$ and $\Delta \neq 0$. So it represent a parabola.

16. (c) : Given equation, $px^3 + qx^2 - rx - r = 0$ having roots α, β, γ. So, we have

$$\alpha+\beta+\gamma = \frac{-q}{p};\ \alpha\beta+\beta\gamma+\gamma\alpha = -\frac{r}{p};\ \alpha\beta\gamma = \frac{r}{p}$$

$$\therefore\ \Delta = \alpha\beta\gamma\left(1+\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right) = \alpha\beta\gamma\left(\frac{\alpha\beta\gamma+\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}\right) = 0$$

17. (b) : $I_{m,n} = \int \cos^m x \cos nx dx$

$$= \frac{\cos^m x \sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x \sin x \sin nx dx$$

$$= \frac{\cos^m x \sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x(\cos(n-1)x - \cos nx \cos x)dx$$

$$I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n}\int \cos^{m-1} x \cos(n-1)\, x dx - \frac{m}{n} I_{m,n}$$

$$\Rightarrow \left(\frac{m+n}{n}\right) I_{m,n} = \frac{\cos^m x \sin nx}{n} + \frac{m}{n} I_{m-1,n-1} + c_1$$

$$\Rightarrow (m+n) I_{m,n} = m I_{m-1,\, n-1} + \cos^m x \sin nx + c$$

18. (d) : We have, $I = \int_0^{\pi/2} \cos 2nx \log \cos x dx$

On integrating by parts, we get,

$$\left|\log \cos x . \frac{\sin 2nx}{2n}\right|_0^{\pi/2} - \int_0^{\pi/2} (-\tan x)\frac{\sin 2nx}{2n} dx$$

Since, $\lim\limits_{x\to\pi/2} \log \cos x . \frac{\sin 2x}{2x} = 0$

$$\therefore \quad I = \int_0^{\pi/2} (\tan x)\frac{\sin 2nx}{2n} dx$$

19. (c) : Since $0 < x < \frac{\pi}{2}$ and $|\cos x| < 1$,

$$y = e^{\frac{\cos^2 x}{1-\cos^2 x}\log_e 2} = e^{\cot^2 x \log_e 2} = 2^{\cot^2 x}$$

Now y satisfies $x^2 - 9x + 8 = 0$

$\therefore \quad y^2 - 9y + 8 = 0 \Rightarrow (y - 1)(y - 8) = 0$

$\Rightarrow \quad y = 1, y = 8$

If $y = 1$ then, $2^{\cot^2 x} = 1 = 2^0$

$\Rightarrow \cot^2 x = 0 \Rightarrow x = \frac{\pi}{2}$ (not possible) $\left[\because\ 0 < x < \frac{\pi}{2}\right]$

If $y = 8$ then $2^{\cot^2 x} = 8 = 2^3 \Rightarrow \cot x = \pm\sqrt{3}$

$\Rightarrow x = \pm\frac{\pi}{6}$ $\left(-\frac{\pi}{6}\right.$ not possible $\because 0 < x < \frac{\pi}{2}$ and $\cot x$ is positive$\left.\right)$

$$\therefore \quad x = \frac{\pi}{6}$$

Now $\frac{\sin x}{\sin x + \cos x} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}+1}$

20. (b) : We have,

$$(2+\sqrt{5})^p + (2-\sqrt{5})^p = 2[2^p + {}^pC_2\, 2^{p-2}\, 5 + {}^pC_4\, 2^{p-4} 5^2 + \ldots + {}^pC_{p-1}\, 2 \cdot 5^{(p-1)/2}] \quad \ldots(1)$$

From (1), $(2+\sqrt{5})^p + (2-\sqrt{5})^p$ is an integer

and $-1 < (2-\sqrt{5})^p < 0$ (as p is odd), so

$$[(2+\sqrt{5})^p] = (2+\sqrt{5})^p + (2-\sqrt{5})^p$$

$$= 2^{p+1} + {}^pC_2\ 2^{p-1}\ 5 + \ldots + {}^pC_{p-1}\ 2^2\ 5^{(p-1)/2}$$

$$\therefore \quad [(2+\sqrt{5})^p] - 2^{p+1} = 2[{}^pC_2\, 2^{p-2}\, 5 + {}^pC_4\, 2^{p-4}\, 5^2 + \ldots + {}^pC_{p-1}\, 2 \cdot 5^{(p-1)/2}]$$

Now, ${}^pC_2 = \frac{p(p-1)}{1 \cdot 2}$, ${}^pC_4 = \frac{p(p-1)(p-2)(p-3)}{1\cdot 2\cdot 3\cdot 4}$,

...., ${}^pC_{p-1} = p$ are divisible by the prime p.

Thus R.H.S. is divisible by p.

21. (a) : We have,

$2a(bc - 4a^2) + b(2ac - b^2) + c(2ab - c^2) = 0$

$\Rightarrow \quad 6abc - 8a^3 - b^3 - c^3 = 0$

$\Rightarrow \quad (2a + b + c)[(2a - b)^2 + (b - c)^2 + (c - 2a)^2] = 0$

$\Rightarrow \quad 2a + b + c = 0 \qquad [\text{as } b \neq c]$

Let $f(x) = 8ax^3 + 2bx^2 + cx$

$$f(0) = 0,\ f\left(\frac{1}{2}\right) = a + \frac{b}{2} + \frac{c}{2} = \frac{2a+b+c}{2} = 0$$

So, $f(x)$ satisfies the Rolle's Theorem condition so, $f'(x) = 0$ has atleast one root in [0, 1/2].

22. (b) : $I = \int \frac{(1-\cos\theta)^{2/7}}{(1+\cos\theta)^{9/7}} d\theta = \frac{1}{2}\int \frac{\left(\sin\frac{\theta}{2}\right)^{4/7}}{\left(\cos\frac{\theta}{2}\right)^{18/7}} d\theta$

Put $\frac{\theta}{2} = t \Rightarrow d\theta = 2dt$

$$\therefore \quad I = \int \frac{(\sin t)^{4/7}}{(\cos t)^{18/7}} dt = \int (\tan t)^{4/7} \sec^2 t dt$$

Put, $\tan t = u \quad \Rightarrow \quad \sec^2 t dt = du$

$$\therefore \quad I = \int u^{4/7} du = \frac{7}{11}\left[\tan\left(\frac{\theta}{2}\right)\right]^{11/7} + c$$

23. (a) : For $x > 0, f(x) = 1 + e^x - x, f'(x) = e^x - 1 = 0$ for $x = 0$ and for $x > 0, f'(x) > 0 \Rightarrow f(x)$ is increasing.

$\Rightarrow f(x) > f(0) \Rightarrow f(x) > 2$. Hence no solution.

Also, curve is symmetrical about y-axis, hence, no solution for $x < 0$ also.

Hence, no solution.

24. (c) : $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{5^n} \cdot {}^nC_r \left(\sum_{t=0}^{r-1} {}^rC_t 3^t \right)$

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{5^n} \cdot {}^nC_r (4^r - 3^r)$$

$$= \lim_{n\to\infty} \frac{1}{5^n} \left(\sum_{r=1}^{n} {}^nC_r 4^r - \sum_{r=1}^{n} {}^nC_r 3^r \right) = \lim_{n\to\infty} \frac{1}{5^n}(5^n - 4^n) = 1$$

25. (a, b, c) : For $a \le 0$ given equation becomes

$$\int_0^2 (x-a)\,dx \ge 1 \Rightarrow a \le \frac{1}{2} \Rightarrow a \le 0$$

For $0 < a < 2$

$$\int_0^2 |x-a|\,dx \ge 1 \Rightarrow \int_0^a (a-x)\,dx + \int_a^2 (x-a)\,dx \ge 1$$

$$\Rightarrow \frac{a^2}{2} + 2 - 2a + \frac{a^2}{2} \ge 1 \Rightarrow a^2 - 2a + 1 \ge 0$$

$\Rightarrow (a - 1)^2 \ge 0.$

For $a \ge 2$, $\int_0^2 |x-a|\,dx \ge 1$

$$\Rightarrow \int_0^2 (a-x)\,dx \ge 1 \Rightarrow 2a - 2 \ge 1 \Rightarrow a \ge \frac{3}{2} \Rightarrow a \ge 2$$

26. (b, c, d) : For a given z, say $z = k$, we have $x, y \le k$, hence there are k^2 ordered pairs (x, y).

So the total number of numbers

$$= \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

27. (a, d) : $\sin\frac{\pi}{24} = \frac{2}{R-2} \Rightarrow R = 2\left(1 + \operatorname{cosec}\frac{\pi}{24}\right)$

(a) is true.

$\because \frac{\left(\sin\frac{\alpha}{2} + \cos\frac{\alpha}{2}\right)^2}{\sin A} = 1 + \operatorname{cosec}\alpha \quad \therefore$ (d) is true.

28. (a, b, c) : $(x^2 + 1) = (x + i)(x - i)$

$f(i) = f(-i)$

$= (\cos\theta + i\sin\theta)^n - (\cos n\theta + i\sin n\theta) = 0$

(By Demoiver theorem)

29. (c, d) : $c + a - b,\ b + c - a,\ a + b - c$ are all positive.

$$\therefore \frac{\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a}}{3} \ge \left[\frac{abc}{(c+a-b)(a+b-c)(b+c-a)}\right]^{1/3} \quad ...(1)$$

Also, $a^2 \ge a^2 - (b - c)^2 \Rightarrow a^2 \ge (a + b - c)(a - b + c)$

Similarly, $b^2 \ge (b + c - a)(b - c + a)$,

$c^2 \ge (c + a - b)(c - a + b)$

$\therefore a^2b^2c^2 \ge (a + b - c)^2(b + c - a)^2(c + a - b)^2$

Thus $abc \ge (a + b - c)(b + c - a)(c + a - b)$

$$\Rightarrow \frac{abc}{(c+a-b)(a+b-c)(b+c-a)} \ge 1$$

So, from (1) $\frac{a}{c+a-b} + \frac{b}{a+b-c} + \frac{c}{b+c-a} \ge 3$

30. (b, c) : We have, $f(x) = x^2 - ax + 2a$

$f'(x) = 2x - a$

At $(2, 4)$, $f'(x) = 4 - a$

$\therefore$ Equation of normal at $(2, 4)$ is

$$(y-4) = -\frac{1}{(4-a)}(x-2)$$

Let point of intersection with x and y axis be A and B respectively, then

$A \equiv (-4a+18, 0)$, $B \equiv \left(0, \frac{4a-18}{a-4}\right)$. Hence $a > \frac{9}{2}$

$\therefore$ Area of triangle $= \frac{1}{2}(4a-18)\frac{(4a-18)}{(a-4)} = 2$

$$\Rightarrow (4a-17)(a-5) = 0 \Rightarrow a = 5 \text{ or } \frac{17}{4}$$

EXAM DATES 2017

SRMJEEE	1st April to 30th April (Online)
JEE MAIN	2nd April (Offline)
	8th & 9th April (Online)
VITEEE	5th April to 16th April (Online)
NATA	16th April
WBJEE	23rd April
Kerala PET	24th April (Physics & Chemistry)
	25th April (Mathematics)
AMU (Engg.)	30th April
Karnataka CET	2nd May (Biology & Mathematics)
	3rd May (Physics & Chemistry)
MHT CET	11th May
COMEDK (Engg.)	14th May
BITSAT	16th May to 30th May (Online)
JEE Advanced	21st May
J & K CET	27th May to 28th May

31. (b, c) :

$$\Delta' x = \begin{vmatrix} 2x+4 & 2x+4 & 13 \\ 4x+5 & 4x+5 & 26 \\ 16x-6 & 16x-6 & 104 \end{vmatrix} + \begin{vmatrix} x^2+4x-3 & 2 & 13 \\ 2x^2+5x-9 & 4 & 26 \\ 8x^2-6x+1 & 16 & 104 \end{vmatrix} = 0$$

$\Rightarrow$ $\Delta(x)$ = constant $\Rightarrow$ $a = 0,\ b = 0,\ c = 0.$

32. (d) : $f(x) = \int_1^x \frac{\ln t}{t(1+t)}dt$

$$f\left(\frac{1}{x}\right) = \int_1^{1/x} \frac{-u\ln u}{\frac{1+u}{u}} \times \frac{-1}{u^2}du = \int_1^x \frac{\ln t}{(1+t)}dt$$

Now $f(x) + f\left(\frac{1}{x}\right) = \int_1^x \frac{\ln t}{(1+t)}\left(1+\frac{1}{t}\right)dt = \int_1^x \frac{\ln t}{t}dt$

$$= \left[\frac{(\ln t)^2}{2}\right]_1^x = \frac{(\ln x)^2}{2}$$

33. (a, b, c) : $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
$- P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$0.4 + 0.6 + 0.5 + 0.75 - (0.4 + 0.6)$
$- P(B \cap C) - 0.35 + 0.2$
[since $P(A \cup B) = P(A) + P(B) - P(A \cap B)$]
$= 1.1 - P(B \cap C)$

But, $0.75 \le P(A \cup B \cup C) \le 1$
$\Rightarrow$ $0.75 \le 1.1 - P(B \cap C) \le 1$
$\Rightarrow$ $0.1 \le P(B \cap C) \le 0.35$

34. (d): $I_1 = \int x\sqrt{a^2 - x^2}dx,$

Put $a^2 - x^2 = t$ $\Rightarrow$ $xdx = -\frac{1}{2}dt$

$\therefore$ $I_1 = -\frac{1}{2}\int \sqrt{t}dt = -\frac{1}{3}t^{3/2} = -\frac{1}{3}(a^2 - x^2)^{3/2} + C$

35. (d): $\int_0^a x^4\sqrt{a^2 - x^2}dx$

$$= \left[\frac{-x^2(a^2 - x^2)^{3/2}}{3}\right]_0^a + \int_0^a x^2(a^2 - x^2)^{3/2}dx$$

$$= \int_0^a (a^2 - x^2)x^2\sqrt{a^2 - x^2}dx$$

$$= \int_0^a a^2\left(x^2\sqrt{a^2 - x^2}\right)dx - \int_0^a x^4\sqrt{a^2 - x^2}dx$$

$$= \frac{a^2}{2}\int_0^a x^2\sqrt{a^2 - x^2}dx \quad \therefore \quad \frac{\int_0^a x^4\sqrt{a^2 - x^2}dx}{\int_0^a x^2\sqrt{a^2 - x^2}dx} = \frac{a^2}{2}$$

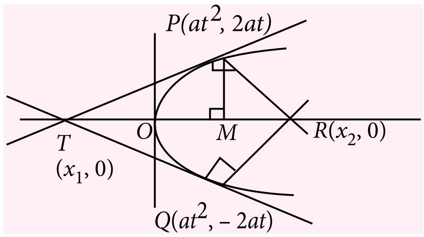
36. (c) : $I_n = \int x^n\sqrt{a^2 - x^2}dx = \int x^{n-1}(x\sqrt{a^2 - x^2})dx$

$$= x^{n-1}\left[-\frac{1}{3}(a^2 - x^2)^{3/2}\right] + \frac{n-1}{3}\int x^{n-2}(a^2 - x^2)\sqrt{a^2 - x^2}dx$$

$$= -\frac{1}{3}x^{n-1}(a^2 - x^2)^{3/2} + \frac{n-1}{3}a^2 I_{n-2} - \frac{n-1}{3}I_n$$

$$\Rightarrow \left(1 + \frac{n-1}{3}\right)I_n = -\frac{1}{3}x^{n-1}(a^2 - x^2)^{3/2} + \frac{a^2(n-1)}{3}I_{n-2}$$

37. (A) - (iv) ; (B) - (ii) ; (C) - (iii) ; (D) - (i)



(A) $y^2 = 4x$ ...(1)

Clearly, if $P(at^2, 2at)$ then by symmetry, $Q(at^2, -2at)$

Equation of tangent is $ty = x + at^2$

For t, $y = 0$, $x_1 = -at^2$ and equation of normal is
$y = -tx + 2at + at^3$

For R, $y = 0$, $x_2 = (2a + at^2)$ ...(2)

Here, $a = 1$ $\Rightarrow x_1 = -t^2$ and $x_2 = (2 + t^2)$

$\Rightarrow$ $3 = 2 + t^2 \Rightarrow t^2 = 1$ $\Rightarrow$ $t = \pm 1$

Take $t = 1$, then $x_1 = -1$.

$\therefore$ $PM = 2at = 2 \times 1 \times 1 = 2$
$RT = x_1 + x_2 = (1 + 3) = 4$

$\therefore$ Area of quadrilateral $PTQR$ = $2 \times \left(\frac{1}{2} \times 4 \times 2\right)$
= 8 sq. units

(B) Length of latus rectum + tangent PT

$$= 4a + \sqrt{S_1} = 4 \times 1 + \sqrt{0^2 - 4(1)} = 4 + 2 = 6 \text{ units}$$

(C) Clearly, RT will be the diameter of circle

$\therefore$ Circumference $= (\pi \times \text{diameter}) = \pi \times RT = \frac{\pi \times 4}{4\pi} = 1$

(D) Since in the part (1), we have found
$x_2 = (2a + at^2) > 2a$, (if $t \neq 0$)

$\therefore$ For three real normals, $x_2 > 2a = 2 \times 1 = 2$

i.e. x_2 = 2 of straight lines otherwise a rectangular hyperbola.

38. (A) - (iii) ; (B) - (iv) ; (C) - (v) ; (D) - (i)

(A) Required area $= 2\int_0^4 \sqrt{x}dx - \frac{1}{2}\times 4\times 4$

$$= \frac{4}{3}\left[x^{3/2}\right]_0^4 - 8 = \frac{8}{3}$$

(B) $\int_0^{\pi}(\sin^5 x + \cos^5 x)dx = 2\int_0^{\pi/2}\sin^5 x dx$

$$= 2\left[\frac{4}{5}\times\frac{2}{3}\right] = \frac{16}{15}$$

(C) For equation $S + k = 0$ to represent pair of lines

$$\begin{vmatrix} 1 & 2 & -2 \\ 2 & 3 & 1 \\ -2 & 1 & 1+k \end{vmatrix} = 0$$

$\Rightarrow\ 3(1+k) - 1 - 2(2+2k+2) - 2(2+6) = 0 \ \Rightarrow\ k = -22$

(D) Let p.v. of given points be $A(\hat{i}+\hat{j}+\hat{k})$, $B(2\hat{i}+2\hat{j}+\hat{k})$ and $C(3\hat{i}-\hat{k})$ so that two vectors in the plane may be $\overrightarrow{AB} = \hat{i}+\hat{j}$ and $\overrightarrow{AC} = 2\hat{i}-\hat{j}-2\hat{k}$

Thus,

$$\begin{vmatrix} \alpha & 2 & 1 \\ 1 & 1 & 0 \\ 2 & -1 & -2 \end{vmatrix} = 0 \Rightarrow -2\alpha - 2(-2) + (-1-2) = 0 \Rightarrow \alpha = \frac{1}{2}$$

39. (0) : Given, $e^{\sin x} - e^{-\sin x} - 4 = 0$

Let $e^{\sin x} = y$ $\quad [y > 0\ \forall\ x \in R]$

then, $y - 1/y - 4 = 0 \Rightarrow y^2 - 4y - 1 = 0$

$$\Rightarrow\quad y = \frac{4\pm\sqrt{16+4}}{2} = 2\pm\sqrt{5}$$

y can never be negative, $2-\sqrt{5}$ can not be accepted.

Now, $2+\sqrt{5} > e$ and maximum value of $e^{\sin x} = e$

Hence, $e^{\sin x} \neq 2+\sqrt{5}$ *i.e.* there is no solution.

40. (9) : Case I : When we use 6, 7 or 8 at ten thousand place then number of numbers $= 3\times {}^4P_4 = 72$

Case II: When we use 5 at ten thousand place and 6, 7 or 8 at thousand place then number of numbers is $= 1\times 3\times {}^3P_3 = 18$

Hence, the required numbers of numbers is

$72 + 18 = 90.$

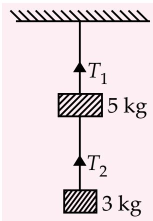
◈◈

FULL LENGTH PRACTICE PAPER BITSAT

Exam date : 16^{th} to 30^{th} May 2017

SECTION-I (PHYSICS)

1. Two masses of 5 kg and 3 kg are suspended with the help of a massless inextensible strings as shown in figure. The whole system is going upwards with an acceleration of 2 m s^{-2}. The tensions T_1 and T_2 are (Take $g = 10$ m s^{-2})



(a) 96 N, 36 N (b) 36 N, 96 N
(c) 96 N, 96 N (d) 36 N, 36 N

2. The dimensions of $\frac{a}{b}$ in the equation, $P = \frac{a - t^2}{bx}$, where P is pressure, x is distance and t is time are

(a) $[M^2LT^{-3}]$ (b) $[ML^0T^{-2}]$
(c) $[ML^3T^{-1}]$ (d) $[MLT^{-3}]$

3. A particle is projected from ground at some angle with the horizontal. Let P be the point at maximum height H. At what height above the point P the particle should be aimed to have range equal to maximum height?

(a) H (b) $2H$ (c) $H/2$ (d) $3H$

4. The dimensions of $\frac{[\text{Angular momentum}]}{[\text{Magnetic moment}]}$ are

(a) $[M^3LT^{-2}A^2]$ (b) $[MA^{-1}T^{-1}]$
(c) $[ML^2A^{-2}T]$ (d) $[M^2L^{-3}AT^2]$

5. A wire of length l and mass m is bent in the form of a rectangle $ABCD$ with $\frac{AB}{BC} = 2$. The moment of inertia of this wire frame about the side BC is

(a) $\frac{11}{252}ml^2$ (b) $\frac{8}{203}ml^2$
(c) $\frac{5}{136}ml^2$ (d) $\frac{7}{162}ml^2$

6. A body is rolling down an inclined plane. If kinetic energy of rotation is 40% of kinetic energy in translatory state, then the body is a

(a) ring (b) cylinder
(c) hollow ball (d) solid ball

7. For two vectors $\vec{A}$ and $\vec{B}$, $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ is always true when

(a) $|\vec{A}| = |\vec{B}| \neq 0$ (b) $\vec{A} \perp \vec{B}$
(c) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are parallel
(d) $|\vec{A}| = |\vec{B}| \neq 0$ and $\vec{A}$ and $\vec{B}$ are antiparallel

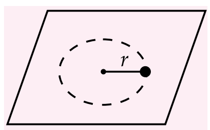
8. An ideal massless spring S can be compressed 2 m by a force of 200 N. This spring is placed at the bottom of the frictionless inclined plane which makes an angle $\theta = 30°$ with the horizontal. A 20 kg mass is released from rest at the top of the inclined plane and is brought to rest momentarily after compressing the spring 4 m. Through what distance does the mass slide before coming to rest?

(a) 2.2 m (b) 4 m (c) 8.17 m (d) 1.9 m

9. Six moles of O_2 gas is heated from 20°C to 35°C at constant volume. If specific heat capacity at constant pressure is 8 cal mol^{-1} K^{-1} and $R = 8.31$ J mol^{-1} K^{-1}. What is the change in internal energy of the gas?

(a) 180 cal (b) 300 cal (c) 360 cal (d) 540 cal

10. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will



(a) decrease by a factor of 2
(b) remain constant
(c) increase by a factor of 2
(d) increase by a factor of 4

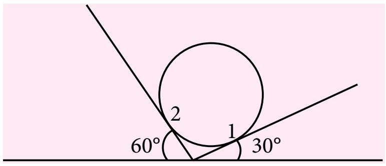
11. A particle of mass m is thrown upwards from the surface of the earth, with a velocity u. The mass and the radius of the earth are, respectively, M and R. G is gravitational constant and g is acceleration due to gravity on the surface of the earth. The minimum

value of u so that the particle does not return back to the earth, is

(a) $\sqrt{\dfrac{2GM}{R^2}}$ (b) $\sqrt{\dfrac{2GM}{R}}$

(c) $\sqrt{\dfrac{2gM}{R^2}}$ (d) $\sqrt{2gR^2}$

12. A solid sphere of mass 10 kg is placed over two smooth inclined planes as shown in figure. Normal reaction at 1 and 2 will be (Take g = 10 m s^{-2})



(a) $50\sqrt{3}$ N, 50 N (b) 50 N, 50 N

(c) 50 N, $50\sqrt{3}$ N (d) 60 N, 40 N

13. A particle of mass M is situated at the centre of a spherical shell of same mass and radius a. The magnitude of the gravitational potential at a point situated at $a/2$ distance from the centre, will be

(a) $\dfrac{GM}{a}$ (b) $\dfrac{2GM}{a}$ (c) $\dfrac{3GM}{a}$ (d) $\dfrac{4GM}{a}$

14. A mass of diatomic gas (γ = 1.4) at a pressure of 2 atmospheres is compressed adiabatically so that its temperature rises from 27°C to 927°C. The pressure of the gas in the final state is

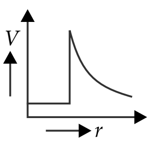
(a) 8 atm (b) 28 atm

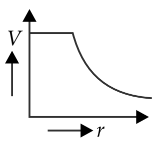
(c) 68.7 atm (d) 256 atm

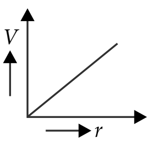
15. If 4 moles of an ideal monatomic gas at temperature 400 K is mixed with 2 moles of another ideal monatomic gas at temperature 700 K, the temperature of the mixture is

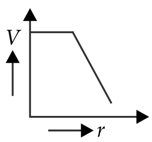
(a) 550°C (b) 500°C (c) 550 K (d) 500 K

16. In the case of a hollow metallic sphere, without any charge inside the sphere, electric potential (V) changes with respect to distance (r) from the centre as

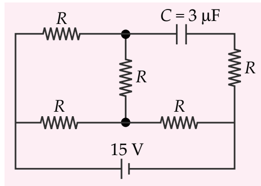
(a) 

(b) 

(c) 

(d) 

17. In the circuit shown, the cell is ideal, with emf = 15 V. Each resistance R is of 3 Ω. The potential difference across the capacitor of capacitance 3 μF is



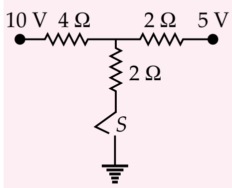
(a) Zero (b) 9 V (c) 12 V (d) 15 V

18. An object of mass 0.2 kg executes simple harmonic oscillations along the x-axis with a frequency $\dfrac{25}{\pi}$ Hz. At the position x = 0.04 m, the object has kinetic energy 0.5 J and potential energy 0.4 J. The amplitude of oscillation is

(Potential energy is zero at mean position)

(a) 6 cm (b) 4 cm (c) 8 cm (d) 2 cm

19. As the switch S is closed in the circuit shown in figure, current passed through it is



(a) Zero

(b) 1 A

(c) 2 A

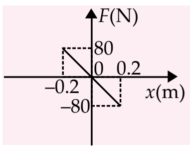
(d) 1.6 A

20. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become

(a) 200 Hz (b) 220 Hz

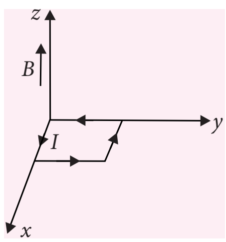
(c) 230 Hz (d) 240 Hz

21. A body of mass 0.01 kg executes simple harmonic motion (SHM) about x = 0 under the influence of a force as shown in the adjacent figure. The period of the SHM is



(a) 1.05 s (b) 0.52 s (c) 0.25 s (d) 0.03 s

22. A uniform magnetic field of 1000 G is established along the positive z-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop as shown in the figure?



(a) Zero (b) 1.8×10^{-2} N m

(c) 1.8×10^{-3} N m (d) 1.8×10^{-4} N m

23. A transverse wave in a medium is described by the equation $y = A\sin 2(\omega t - kx)$. The magnitude of the maximum velocity of particles in the medium is equal to that of the wave velocity, if the value of A is

(a) $\frac{\lambda}{2\pi}$ (b) $\frac{\lambda}{4\pi}$ (c) $\frac{\lambda}{\pi}$ (d) $\frac{2\lambda}{\pi}$

24. A small square loop of wire of side l is placed inside a large square loop of wire of side L ($>> l$). The loops are coplanar and their centres coincide. What is the mutual inductance of the system?

(a) $2\sqrt{2}\frac{\mu_0}{\pi}\frac{l^2}{L}$ (b) $8\sqrt{2}\frac{\mu_0}{\pi}\frac{l^2}{L}$

(c) $2\sqrt{2}\frac{\mu_0}{2\pi}\frac{l^2}{L}$ (d) None of these

25. An unpolarized light beam is incident on a surface at an angle of incidence equal to Brewster's angle. Then,

(a) the reflected and the refracted beams are both partially polarized

(b) the reflected beam is partially polarized and the refracted beam is completely polarized and are at right angles to each other

(c) the reflected beam is completely polarized and the refracted beam is partially polarized and are at right angles to each other

(d) both the reflected and the refracted beams are completely polarized and are at right angles to each other.

26. The range of voltmeter of resistance 300 Ω is 5 V. The resistance to be connected to convert it into an ammeter of range 5 A is

(a) 1 Ω in series (b) 1 Ω in parallel

(c) 0.1 Ω in series (d) 0.1 Ω in parallel

27. A square frame of side 1 m carries a current I, produces a magnetic field B at its centre. The same current is passed through a circular coil having the same perimeter as the square. The magnetic field at the centre of the circular coil is B'. The ratio B/B' is

(a) $\frac{8}{\pi^2}$ (b) $\frac{8\sqrt{2}}{\pi^2}$ (c) $\frac{16}{\pi^2}$ (d) $\frac{16}{\sqrt{2}\pi^2}$

28. A conducting circular loop is placed in a uniform magnetic field, B = 0.025 T with its plane perpendicular to the loop. The radius of the loop is made to shrink at a constant rate of 1 mm s^{-1}. The induced emf when the radius is 2 cm, is

(a) 2π μV (b) π μV (c) $\frac{\pi}{2}$ μV (d) 2 μV

29. In an ac circuit, the current is $i = 5\sin\left(100t - \frac{\pi}{2}\right)$ ampere and the potential difference is $V = 200\sin(100t)$ volt. Then the power consumed is

(a) 200 W (b) 500 W (c) 1000 W (d) Zero

30. The resistance of the wire in the platinum resistance thermometer at ice point is 5 Ω and at steam point is 5.25 Ω. When the thermometer is inserted in an unknown hot bath its resistance is found to be 5.5 Ω. The temperature of the hot bath is

(a) 100°C (b) 200°C (c) 300°C (d) 350°C

31. A series LCR circuit is connected to an ac source of variable frequency. When the frequency is increased continuously, starting from a small value, the power factor

(a) goes on increasing continuously

(b) goes on decreasing continuously

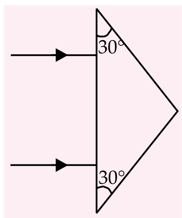
(c) becomes maximum at a particular frequency

(d) remains constant

32. A coil has 1000 turns and 500 cm^2 as its area. The plane of the coil is placed perpendicular to a uniform magnetic field of 2×10^{-5} T. The coil is rotated through 180° in 0.2 seconds. The average emf induced in the coil, in mV is

(a) 5 (b) 10 (c) 15 (d) 20

33. Two parallel light rays are incident at one surface of a prism as shown in the figure. The prism is made of glass of refractive index 1.5. The angle between the rays as they emerge is nearly



(a) 19° (b) 37° (c) 45° (d) 49°

34. What is the conductivity of a semiconductor if electron density = 5×10^{12} cm^{-3} and hole density = 8×10^{13} cm^{-3}?
(μ_e = 2.3 V^{-1} s^{-1} m^2 and μ_h = 0.01 V^{-1} s^{-1} m^2)

(a) 5.634 Ω^{-1} m^{-1} (b) 1.968 Ω^{-1} m^{-1}

(c) 3.421 Ω^{-1} m^{-1} (d) 8.964 Ω^{-1} m^{-1}

35. If λ_1 and λ_2 are the wavelengths of the first members of the Lyman and Paschen series respectively, then $\lambda_1 : \lambda_2$ is

(a) 1 : 3 (b) 1 : 30 (c) 7 : 50 (d) 7 : 108

36. The half life of radioactive radon is 3.8 days. The time at the end of which $\left(\frac{1}{20}\right)^{\text{th}}$ of the radon sample will remain undecayed is

(a) 3.8 days (b) 16.5 days
(c) 33 days (d) 76 days

37. A hydrogen atom and a Li^{2+} ion are both in the second excited state. If l_H and l_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then

(a) $l_H = l_{Li}$ and $|E_H| < |E_{Li}|$

(b) $l_H > l_{Li}$ and $|E_H| > |E_{Li}|$

(c) $l_H = l_{Li}$ and $|E_H| > |E_{Li}|$

(d) $l_H < l_{Li}$ and $|E_H| < |E_{Li}|$

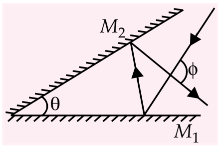
38. The ratio of de Broglie wavelength of a proton and an α particle accelerated through the same potential difference is

(a) $3\sqrt{2}$ (b) $2\sqrt{2}$ (c) $2\sqrt{3}$ (d) $2\sqrt{5}$

39. The temperature of the sink of a Carnot engine is 27°C and its efficiency is 25%. The temperature of the source is

(a) 227°C (b) 27°C (c) 327°C (d) 127°C

40. The reflecting surfaces of two mirrors M_1 and M_2 are at an angle θ (angle θ between 0° and 90°) as shown in the figure. A ray of light is incident on M_1. The emerging ray intersects the incident ray at an angle ϕ. Then,



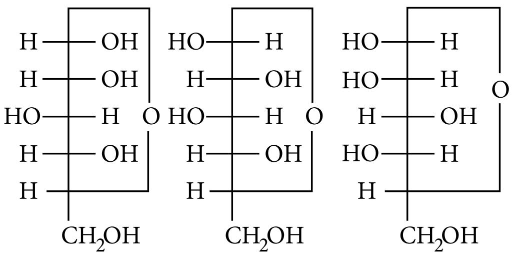
(a) $\phi = \theta$ (b) $\phi = 180° - \theta$
(c) $\phi = 90° - \theta$ (d) $\phi = 180° - 2\theta$

SECTION-II (CHEMISTRY)

41. A nuclide of an alkaline earth metal undergoes radioactive decay by emission of α-particle in succession to give the stable nucleus. The group of the periodic table to which the resulting daughter element would belong is

(a) Group 4 (b) Group 6
(c) Group 16 (d) Group 14

42. Three cyclic structures of monosaccharides are given below. Which of these are anomers?



(I) (II) (III)

(a) I and II (b) II and III
(c) I and III
(d) III is anomer of I and II.

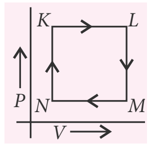
43. The electronegativity of the following elements increases in the order

(a) C < N < Si < P (b) N < Si < C < P
(c) Si < P < C < N (d) P < Si < N < C

44. The atomic masses of He and Ne are 4 and 20 amu respectively. The value of the de Broglie wavelength of He gas at –73°C is 'M' times that of the de Broglie wavelength of Ne at 727°C. 'M' is

(a) 2 (b) 3 (c) 4 (d) 5

45. A fixed mass of a gas is subjected to transformations of state from K to L to M to N and back to K as shown. The pair of isochoric processes among the transformations of state is



(a) K to L and L to M
(b) L to M and N to K
(c) L to M and M to N
(d) M to N and N to K

46. For the estimation of nitrogen, 1.4 g of an organic compound was digested by Kjeldahl's method and the evolved ammonia was absorbed in 60 mL of $\frac{M}{10}$ sulphuric acid. The unreacted acid required 20 mL of $\frac{M}{10}$ sodium hydroxide for complete neutralisation. The percentage of nitrogen in the compound is

(a) 5% (b) 6% (c) 10% (d) 3%

47. Which one of the following sequences represents the correct increasing order of bond angles in the given molecules?

(a) $H_2O < OF_2 < OCl_2 < ClO_2$
(b) $OCl_2 < ClO_2 < H_2O < OF_2$
(c) $OF_2 < H_2O < OCl_2 < ClO_2$
(d) $ClO_2 < OF_2 < OCl_2 < H_2O$

48. Which one of the following is the correct statement?

(a) $B_2H_6 \cdot 2NH_3$ is known as 'inorganic benzene'.
(b) Boric acid is a protonic acid.
(c) Beryllium exhibits coordination number of six.
(d) Chlorides of both beryllium and aluminium have bridged chloride structures in solid phase.

49. van der Waals' equation for 0.2 mol of a gas is

(a) $\left(P+\frac{a}{V^2}\right)+(V-0.2b)=0.2RT$

(b) $\left(P+\frac{a}{0.04V^2}\right)(V-b)=0.02RT$

(c) $\left(P+\frac{0.2a}{V^2}\right)(V-0.2b)=0.2RT$

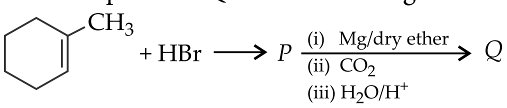
(d) $\left(P+\frac{0.04a}{V^2}\right)(V-0.2b)=0.2RT$

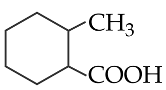
50. If λ_0 is the threshold wavelength for photoelectric emission, λ the wavelength of light falling on the surface of a metal and m the mass of the electron, then the velocity of ejected electron is given by

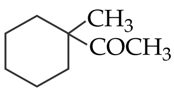
(a) $\left[\frac{2h}{m}(\lambda_0-\lambda)\right]^{1/2}$ (b) $\left[\frac{2hc}{m}(\lambda_0-\lambda)\right]^{1/2}$

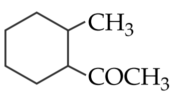
(c) $\left[\frac{2hc}{m}\left(\frac{\lambda_0-\lambda}{\lambda_0\lambda}\right)\right]^{1/2}$ (d) $\left[\frac{2h}{m}\left(\frac{1}{\lambda_0}-\frac{1}{\lambda}\right)\right]^{1/2}$

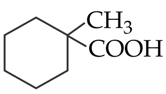
51. The final product 'Q' in the following reaction is



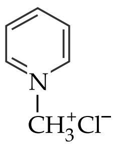
(a) 

(b) 

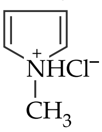
(c) 

(d) 

52. Which one of the following is a quaternary salt?

(a) 

(b) $(CH_3)_3\overset{+}{N}HCl^-$

(c) 

(d) $(CH_3)_4\overset{+}{N}Cl^-$

53. Silver is monovalent and has an atomic mass of 108. Copper is divalent and has an atomic mass of 63.6. The same electric current is passed, for the same length of time through a silver coulometer and a copper coulometer. If 27.0 g of silver is deposited, then the corresponding amount of copper deposited is

(a) 63.60 g (b) 31.80 g (c) 15.90 g (d) 7.95 g

54. A compound contains atoms X, Y and Z. The oxidation number of X is +3, Y is +5 and Z is –2. The possible formula of the compound is

(a) XYZ_2 (b) $Y_2(XZ_3)_2$

(c) $X_3(YZ_4)_3$ (d) $X_3(Y_4Z)_2$

55. Which of the following values of stability constant K corresponds to the most unstable complex compound?

(a) 1.6×10^7 (b) 4.5×10^{14}

(c) 2.0×10^{27} (d) 5.0×10^{33}

56. 0.22 g of an organic compound C_xH_yO which occupied 112 mL at STP, and on combustion gave 0.44 g of CO_2. The ratio of x to y in the compound is

(a) 1 : 1 (b) 1 : 2 (c) 1 : 3 (d) 1 : 4

57. Which of the following is pseudo alum?

(a) $(NH_4)_2SO_4.Fe_2(SO_4)_3.24H_2O$

(b) $K_2SO_4.Al_2(SO_4)_3.24H_2O$

(c) $MnSO_4.Al_2(SO_4)_3.24H_2O$

(d) None of these.

58. Which of the following is not the characteristic property of alcohols?

(a) Lower members have pleasant smell, higher members are odourless, tasteless.

(b) Lower members are insoluble in water, but it regularly increases with increase in molecualar weight.

(c) Boiling point rises regularly with the increase in molecular weight.

(d) Alcohols are lighter than water.

59. For the reaction $A + 2B \rightleftharpoons C + D$, the equilibrium constant is 1.0×10^8. Calculate the equilibrium concentration of A if 1.0 mole of A and 3.0 moles of B are placed in 1 L flask and allowed to attain the equilibrium.

(a) 1.0×10^{-2} mol L^{-1} (b) 2.1×10^{-4} mol L^{-1}

(c) 5×10^{-5} mol L^{-1} (d) 1.0×10^{-8} mol L^{-1}

60. The correct order of basicities of the following compounds is

$CH_3-C(=NH)-NH_2$ (I) $CH_3-CH_2-NH_2$ (II)

$(CH_3)_2NH$ (III) $CH_3-C(=O)-NH_2$ (IV)

(a) II > I > III > IV (b) I > III > II > IV

(c) III > I > II > IV (d) I > II > III > IV

61. $CH_3-\overset{CH_3}{\underset{CH_3}{C}}-CH=CH_2 \xrightarrow[H_2O_2,\ OH^-]{B_2H_6,\ THF} (A)$

The product (A) is

(a) $CH_3-\overset{CH_3}{\underset{CH_3}{C}}-\underset{OH}{CH}-CH_3$

(b) $CH_3-\overset{OH}{\underset{CH_3}{C}}-\underset{CH_3}{CH}-CH_3$

(c) $CH_3-\overset{CH_3}{\underset{CH_3}{C}}-CH_2-CH_2OH$

(d) $\underset{OH}{CH_2}-\overset{CH_3}{\underset{CH_3}{C}}-CH_2-CH_3$

62. Which one of the following statements is not true?

(a) For first order reaction, straight line graph of log $(a - x)$ versus t is obtained for which slope = $- k/2.303$.

(b) A plot of log k *vs* $1/T$ gives a straight line graph for which slope = $- E_a/2.303R$.

(c) For third order reaction, the product of $t_{1/2}$ and initial concentration a is constant.

(d) Units of k for the first order reaction are independent of concentration units.

63. If K_1 and K_2 are the ionization constants of $H_3\overset{+}{N}CHRCOOH$ and $H_3\overset{+}{N}CHRCOO^-$ respectively, the pH of the solution at the isoelectric point is

(a) $pH = pK_1 + pK_2$ (b) $pH = (pK_1\ pK_2)^{1/2}$

(c) $pH = (pK_1 + pK_2)^{1/2}$ (d) $pH = (pK_1 + pK_2)/2$

64. The rate of change of concentration of (A) for reaction: $A \longrightarrow B$ is given by

$$\frac{-d[A]}{dt} = k[A]^{1/3}$$

The half-life period of the reaction will be

(a) $\dfrac{3[A_0]^{2/3}[(2)^{2/3}-1]^2}{(2)^{5/3}k}$ (b) $\dfrac{\frac{3}{2}[A_0]^{2/3}[(2)^{2/3}-1]}{k}$

(c) $\dfrac{3[A_0]^{2/3}[(2)^{2/3}-1]}{(2)^{5/3}k}$ (d) $\dfrac{\frac{2}{3}[A_0]^{3/2}[(2)^{2/3}-1]}{k}$

65. Which of the following compounds have two lone pairs of electrons?

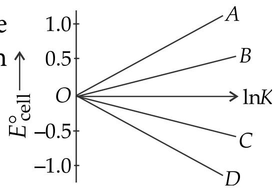
NO_2^-, OF_2, XeF_2, ICl_3
I II III IV

(a) I and III (b) II and III

(c) II and IV (d) I and IV

66. Given: $\Delta G° = -nFE°_{cell}$ and $\Delta G° = - RT\ln K$

The value of $n = 2$ will be given by the slope of which line in the given figure?



(a) OA

(b) OB

(c) OC (d) OD

67. An amide, C_3H_7NO upon acid hydrolysis gives an acid, $C_3H_6O_2$ which on reaction with Br_2/P gives α-bromo acid which when boiled with aq. OH^- followed by acidification gives lactic acid. What is the structural formula of amide?

(a) $CH_3-\overset{O}{\overset{\|}{C}}-NH-CH_3$ (b) $H-\overset{O}{\overset{\|}{C}}-NH-CH_2CH_3$

(c) $CH_3-CH_2-\overset{O}{\overset{\|}{C}}-NH_2$

(d) $H-\overset{O}{\overset{\|}{C}}-N(CH_3)_2$

68. Certain volume of a gas exerts some pressure on walls of a container at constant temperature. It has been found that by reducing the volume of the gas to half of its original value, the pressure becomes twice that of initial value at constant temperature. This is because

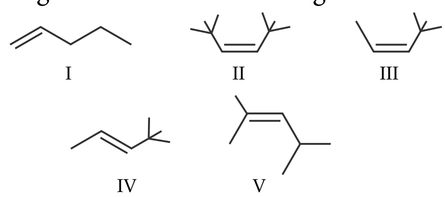
(a) the weight of the gas increases with pressure

(b) velocity of gas molecules decreases

(c) more number of gas molecules strike the surface per second

(d) gas molecules attract one another.

69. Choose the correct comparison of heat of hydrogenation for the following alkenes:



(a) II < IV < III < V < I

(b) III < IV < I < V < II

(c) V < IV < III < I < II

(d) IV < V < I < III < II

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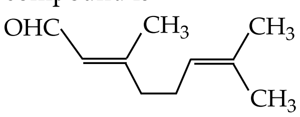
70. The data for the reaction: $A + B \rightarrow C$, is given below:

Ex.	$[A]_0$	$[B]_0$	Initial rate
1.	0.012	0.035	0.10
2.	0.024	0.070	0.80
3.	0.024	0.035	0.10
4.	0.012	0.070	0.80

The rate law which corresponds to the above data is
(a) rate = $k[B]^3$ (b) rate = $k[B]^4$
(c) rate = $k[A][B]^3$ (d) rate = $k[A]^2[B]^2$

71. Which one of the following octahedral complexes will not show geometrical isomerism?
(A and B are monodentate ligands.)
(a) $[MA_2B_2]$ (b) $[MA_3B_3]$
(c) $[MA_4B_2]$ (d) $[MA_5B]$

72. The correct IUPAC name of the following compound is



(a) 3,7,7-trimethylhepta-2,6-dien-1-al
(b) 2,6-dimethylocta-3,7-dien-1-al
(c) 2,6,6-trimethylhepta-3,7-dien-1-al
(d) 3,7-dimethylocta-2,6-dien-1-al.

73. Catenation, *i.e.*, linking of similar atoms depends on size and electronic configuration of atoms. The tendency of catenation in group 14 elements follows the order
(a) C > Si > Ge > Sn (b) C >> Si > Ge ≈ Sn
(c) Si > C > Sn > Ge (d) Ge > Sn > Si > C

74. In a solid, oxide ions are arranged in *ccp*. One-sixth of the tetrahedral voids are occupied by cations X while one-third of the octahedral voids are occupied by the cations Y. The formula of the compound is
(a) X_2YO_3 (b) XYO_3 (c) XY_2O_3 (d) $X_2Y_2O_3$

75. In an atom, an electron is moving with a speed of 600 m s^{-1} with an accuracy of 0.005%. Certainty with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg m^2 s^{-1}, mass of electron, $m_e = 9.1 \times 10^{-31}$ kg)
(a) 1.52×10^{-4} m (b) 5.10×10^{-3} m
(c) 1.92×10^{-3} m (d) 3.84×10^{-3} m

76. Which of the following cannot be prepared by Williamson's synthesis?
(a) Methoxybenzene
(b) Benzyl *p*-nitrophenyl ether
(c) Methyl *tert.*-butyl ether
(d) Di-*tert.*-butyl ether

77. Which of the following reactions is an example of S_N2 reaction?
(a) $CH_3Br + OH^- \longrightarrow CH_3OH + Br^-$
(b) $CH_3{-}\underset{\displaystyle Br}{\underset{|}{C}H}{-}CH_3 + OH^- \longrightarrow CH_3{-}\underset{\displaystyle OH}{\underset{|}{C}H}{-}CH_3$
(c) $CH_3CH_2OH \xrightarrow{-H_2O} CH_2{=}CH_2$
(d) $(CH_3)_3C{-}Br + OH^- \longrightarrow (CH_3)_3C{-}OH + Br^-$

78. The solubility of metal halides depends on their nature, lattice enthalpy and hydration enthalpy of the individual ions. Amongst fluorides of alkali metals, the lowest solubility of LiF in water is due to
(a) ionic nature of lithium fluoride
(b) high lattice enthalpy
(c) high hydration enthalpy for lithium ion
(d) low ionisation enthalpy of lithium atom.

79. Which of the following statements is incorrect?
(a) The covalent bonds have some partial ionic character.
(b) The ionic bonds have some partial covalent character.
(c) The larger the size of the cation and the smaller the size of the anion, the greater is the covalent character of the ionic bond.
(d) The greater the charge on the cation, the greater is the covalent character of the ionic bond.

80. Which of the following is a constituent of nylon?
(a) Adipic acid (b) Styrene
(c) Teflon (d) None of these

SECTION-III (ENGLISH AND LOGICAL REASONING)

PASSAGE

Direction (Questions 81 to 84): Read the passage and answer the following questions.

Democratic societies from the earliest times have expected their governments to protect the weak against the strong. No 'era of good feeling' can justify discharging the police force or giving up the idea of public control over concentrated private wealth. On the other hand, it is obvious that a spirit of self-denial and moderation on the part of those who hold economic power will greatly soften the demand for absolute equality. Men are more interested in freedom and security than in an

equal distribution of wealth. The extent to which the government must interfere with business, therefore, is not exactly measured by the extent to which economic power is concentrated into a few hands. The required degree of government interference depends mainly on whether economic powers are oppressively used, and on the necessity of keeping economic factors in a tolerable state of balance.

But with the necessity of meeting all these dangers and threats to liberty, the powers of the government are unavoidably increased, whichever political party may be in office. The growth of government is a necessary result of the growth of technology and of the problems that go with the use of machines and science. Since the government in our nation, must take on more powers to meet its problems, there is no way to preserve freedom except by making democracy more powerful.

81. The advent of science and technology has increased the ______.
(a) powers of the governments
(b) chances of economic inequality
(c) freedom of people
(d) tyranny of the political parties

82. A spirit of moderation on the economically sound people would make the less privileged ______.
(a) unhappy with their lot
(b) clamour less for absolute equality
(c) unhappy with the rich people
(d) more interested in freedom and security

83. The growth of government is necessitated to ______.
(a) monitor science and technology
(b) deploy the police force wisely
(c) make the rich and the poor happy
(d) curb the accumulation of wealth in a few hands

84. 'Era of good feeling' in sentence 2 refers to ______.
(a) time without government
(b) time of police atrocities
(c) time of prosperity
(d) time of adversity

Direction (Questions 85 to 86): Choose the correct antonym for each of the following words.

85. **Autonomy**
(a) Subordination (b) Slavery
(c) Submissiveness (d) Dependence

86. **Laconic**
(a) Prolific (b) Bucolic
(c) Prolix (d) Profligate

Direction (Questions 87 to 89): In each of the following questions. Find out which part of the sentence has an error. If there is no mistake, the answer is 'No error'.

87. (a) Having deprived from their homes
(b) in the recent earthquake
(c) they had no other option but
(d) to take shelter in a school.

88. (a) The Ahujas
(b) are living in this colony
(c) for the last eight years.
(d) No error.

89. (a) She sang (b) very well
(c) isn't it? (d) No error.

Direction (Questions 90 to 93): Rearrange the given five sentences A, B, C, D, E in the proper sequence so as to form a meaningful paragraph and then answer the given questions.

A. Many consider it wrong to blight youngsters by recruiting them into armed forces at a young age.
B. It is very difficult to have an agreement on an issue when emotions run high.
C. The debate has again come up whether this is right or wrong.
D. In many countries military service is compulsory for all.
E. Some of these detractors of compulsory draft are even very angry.

90. Which sentence should come fourth in the paragraph?
(a) A (b) B (c) C (d) D

91. Which sentence should come first in the paragraph?
(a) A (b) B (c) C (d) D

92. Which sentence should come last in the paragraph?
(a) A (b) B (c) C (d) D

93. Which sentence should come third in the paragraph?
(a) A (b) B (c) C (d) E

Direction (Questions 94 to 95): In each of the following questions, a word has been written in four different ways out of which only one is correctly spelt. Choose the correctly spelt word.

94. (a) Sepalchral (b) Sepulchrle
(c) Sepulchral (d) Sepalchrle

95. (a) Overleped (b) Overelaped
(c) Overlapped (d) Overlaped

96. How many such pairs of letters are there in the word RECRUIT each of which has as many letters between them in the word as they have in the English alphabet series?
(a) None (b) One (c) Two (d) Three

97. Which of the following forms the mirror image of given word, if the mirror is placed vertically to the left?

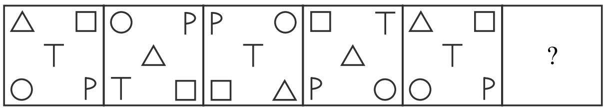
1 B F 9 8 1 6 i F

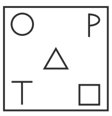
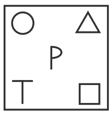
(a) I B Ⅎ ∂ 8 I ɘ ! Ⅎ (b) ꟻ i ∂ I 8 ɘ ꟻ ᗺ I
(c) I i ∂ I 8 ɘ ꟻ ᗺ ꟻ (d) I ! 9 [8 6 Ⅎ ᗺ Ⅎ

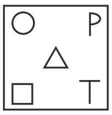
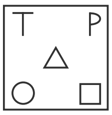
98. In a certain code language,
I. 'she likes apples' is written as 'pic sip dip'.
II. 'parrot likes apples lots' is written as 'dip pic tif nif'.
III. 'she likes parrot' is written as 'tif sip dip'.
How is 'parrot' written in that code language?
(a) pic (b) dip (c) tif (d) nif

99. Find the figure from the options which will continue the series established by problem figures.

Problem Figures



(a)  (b) 

(c)  (d) 

100. X's mother is the mother-in-law of the father of Z. Z is the brother of Y while X is the father of M. How is X related to Z?
(a) Paternal uncle (b) Maternal uncle
(c) Cousin (d) Grandfather

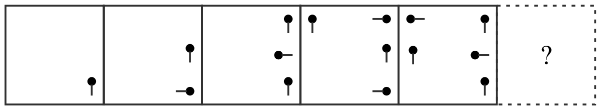
101. Read the following information carefully and answer the question given below it :
1. There is a group of six persons A, B, C, D, E and F in a family. They are Psychologist, Manager, Lawyer, Jeweller, Doctor and Engineer.
2. The doctor is the grandfather of F who is a Psychologist.
3. The Manager D is married to A.
4. C, the Jeweller, is married to the Lawyer.
5. B is the mother of F and E.
6. There are two married couples in the family.

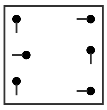
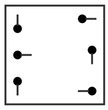
What is the profession of E?
(a) Doctor (b) Engineer
(c) Manager (d) Psychologist

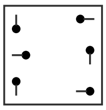
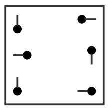
102. Study the information given below carefully and answer the question that follows.
On a playing ground, Mohit, Kartik, Nitin, Atul and Pratik are standing as described below facing the North.
I. Kartik is 40 metres to the right of Atul.
II. Mohit is 70 metres to the south of Kartik.
III. Nitin is 35 metres to the west of Atul.
IV. Pratik is 100 metres to the north of Mohit.
Who is to the north-east of the person who is to the left of Kartik?
(a) Mohit (b) Nitin (c) Pratik (d) Atul

103. Select a figure from the options which will continue the same series as established by the Problem Figures.

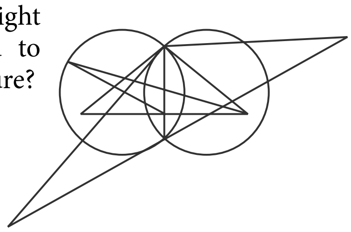
Problem Figures



(a)  (b) 

(c)  (d) 

104. How many straight lines are required to draw the given figure?



(a) 8
(b) 9
(c) 10
(d) 11

105. Select a figure from the options which forms the water image of the given combination of letters/numbers.

8C185e2F9

(a) 6ꟻ2ɘ581Ↄ8 (b) 8C185ɘ2Ⅎ∂
(c) 8Ↄ1 85ɘ2Ⅎ∂ (d) ɘꟻ2ɘ581Ↄ8

SECTION-IV (MATHEMATICS)

106. Three numbers, the third of which being 12, form decreasing G.P. If the last term was 9 instead of 12, the three numbers would have formed an A.P. The common ratio of the G.P. is
(a) 1/3 (b) 2/3 (c) 3/4 (d) 4/5

107. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the

point B. Besides, no three lines pass through one point, no lines passes through both points A and B, and no two are parallel, then the number of intersection points the lines have is equal to

(a) 535 (b) 601 (c) 728 (d) 963

108. The value of the determinant

$$\begin{vmatrix} 1 & e^{i\pi/3} & e^{i\pi/4} \\ e^{-i\pi/3} & 1 & e^{2i\pi/3} \\ e^{-i\pi/4} & e^{-2i\pi/3} & 1 \end{vmatrix}, \text{where } i = \sqrt{-1}, \text{ is}$$

(a) $2+\sqrt{2}$ (b) $-(2+\sqrt{2})$

(c) $-2+\sqrt{3}$ (d) $-2-\sqrt{3}$

109. A rifle man firing at a distant target and has only 10% chance of hitting it. The minimum number of rounds he must fire in order to have 50% chance of hitting it atleast once is

(a) 6 (b) 7 (c) 8 (d) 9

110. $\lim\limits_{x\to 0} \dfrac{1-\cos x \cos 2x \cos 3x}{\sin^2 2x}$ is equal to

(a) 7/2 (b) 7/3 (c) 7/4 (d) 7/5

111. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

112. The points A, B and C represent the complex numbers z_1, z_2, $(1 - i)z_1 + iz_2$ (where $i = \sqrt{-1}$) respectively on the complex plane. The triangle ABC is

(a) isosceles but not right angled

(b) right angled but not isosceles

(c) isosceles and right angled

(d) none of these

113. The area of the figure bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(a) 2 sq. units (b) 3 sq. units

(c) 4 sq. units (d) 1 sq. unit

114. The direction cosines of the line drawn from $P(-5, 3, 1)$ to $Q(1, 5, -2)$ is

(a) $(6, 2, -3)$ (b) $(2, -4, 1)$

(c) $(-4, 8, -1)$ (d) $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

115. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$ is equal to

(a) 1/2 (b) $\cos \pi/8$

(c) 1/8 (d) $\dfrac{1+\sqrt{2}}{2\sqrt{2}}$

116. The two vectors $\{\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \vec{b} = 4\hat{i} - \lambda\hat{j} + 6\hat{k}\}$ are parallel if λ is equal to

(a) 2 (b) -3 (c) 3 (d) -2

117. Let $f(x) = \begin{cases} \dfrac{a|x^2 - x - 2|}{2 + x - x^2}, & x < 2 \\ b, & x = 2 \\ \dfrac{x - [x]}{x - 2}, & x > 2 \end{cases}$

([·] denotes the greatest integer function)

If $f(x)$ is continuous at $x = 2$, then

(a) $a = 1, b = 2$ (b) $a = 1, b = 1$

(c) $a = 0, b = 1$ (d) $a = 2, b = 1$

118. Let $a, b, c \in R$ and $a \neq 0$. If α is a root of $a^2x^2 + bx + c = 0$, β is a root of $a^2x^2 - bx - c = 0$ and $0 < \alpha < \beta$, then the equation $a^2x^2 + 2bx + 2c = 0$ has a root γ that always satisfies

(a) $\gamma = \alpha$ (b) $\gamma = \beta$

(c) $\gamma = (\alpha + \beta)/2$ (d) $\alpha < \gamma < \beta$

119. The term independent of x in the expansion of $\left[\sqrt{\left(\frac{x}{3}\right)} + \sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is

(a) 5/12 (b) 1

(c) 1/3 (d) None of these

120. If $a < 0$, the function $f(x) = e^{ax} + e^{-ax}$ is a monotonically decreasing function for values of x given by

(a) $x > 0$ (b) $x < 0$

(c) $x > 1$ (d) $x < 1$

121. The number of solutions of the equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

122. Let R be a relation on the set of all lines in a plane defined by $(l_1, l_2) \in R$ such that $l_1 \parallel l_2$ then R is

(a) reflexive only (b) symmetric only

(c) transitive only (d) equivalence

123. Let $P(n) = 5^n - 2^n$, $P(n)$ is divisible by 3λ, where λ and n both are odd positive integers then the least value of n and λ will be

(a) 13 (b) 11 (c) 1 (d) 5

124. The rank of $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to

(a) 4 (b) 3 (c) 5 (d) 1

125. A single letter is selected at random from the word 'PROBABILITY'. The probability that it is a vowel, is

(a) 2/11 (b) 3/11
(c) 4/11 (d) none of these

126. Let $A \equiv \{1, 2, 3, 4\}$, $B \equiv \{a, b, c\}$, then number of functions from $A \to B$, which are not onto is

(a) 8 (b) 24 (c) 45 (d) 6

127. The minimum value of the function defined by $f(x) = \text{maximum } \{x, x+1, 2-x\}$ is

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

128. For parabola $x^2 + y^2 + 2xy - 6x - 2y + 3 = 0$, the focus is

(a) $(1, -1)$ (b) $(-1, 1)$
(c) $(3, 1)$ (d) none of these

129. If $A = \{x : x = 4n + 1, \forall\ 2 \le n \le 6\}$, then number of subsets of A are

(a) 2^2 (b) 2^3 (c) 2^5 (d) 2^6

130. The mean deviation and S.D. about actual mean of the series $a, a + d, a + 2d, \ldots, a + 2nd$ are respectively

(a) $\frac{n(n+1)d}{2n+1}, \sqrt{\frac{n(n-1)}{3}} \cdot d$

(b) $\frac{n(n-1)}{3}, \frac{n(n+1)}{2n} \cdot d$

(c) $\frac{n(n+1)d}{(2n+1)}, \sqrt{\frac{n(n+1)}{3}} \cdot d$

(d) $\frac{n(n-1)d}{2n-1}, \sqrt{\frac{n(n-1)}{3}} \cdot d$

131. If the arithmetic progression whose common difference is non zero, the sum of first $3n$ terms is equal to the sum of the next n terms. Then the ratio of the sum of the first $2n$ terms to the next $2n$ terms is

(a) 1 : 5 (b) 2 : 3
(c) 3 : 4 (d) none of these

132. $\lim_{n \to \infty} \frac{(n!)^{1/n}}{n}$ equals

(a) e (b) e^{-1}
(c) 1 (d) none of these

133. A man of height 2 m walks directly away from a lamp of height 5 m, on a level road at 3 m/s. The rate at which the length of his shadow is increasing is

(a) 1 m/s (b) 2 m/s (c) 3 m/s (d) 4 m/s

134. If the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide, then the value of b^2 is

(a) 3 (b) 16 (c) 9 (d) 12

135. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) π

136. If the sum of the binomial coefficients in the expansion of $\left(x + \frac{1}{x}\right)^n$ is 64, then the term independent of x is equal to

(a) 10 (b) 20 (c) 40 (d) 60

137. Solution of the differential equation $\sin y \frac{dy}{dx} = \cos y(1 - x \cos y)$ is

(a) $\sec y = x - 1 - ce^x$ (b) $\sec y = x + 1 + ce^x$
(c) $\sec y = x + e^x + c$ (d) none of these

138. The function $f(x) = |x^2 - 3x + 2| + \cos|x|$ is not differentiable at x is equal to

(a) -1 (b) 0 (c) 1 (d) 2

139. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. Then the number of such arrangements is

(a) at least 500 but less than 750
(b) at least 750 but less than 1000
(c) at least 1000 (d) less than 500

140. A normal to $y^2 = 4ax$ at t touches $x^2 - y^2 = a^2$, then $(t^2 + 1)^3$ is

(a) < 0 (b) > 0
(c) ≤ 0 (d) nothing can be said

141. If $f : X \to Y$ defined by $f(x) = \sqrt{3} \sin x + \cos x + 4$ is one-one and onto, then Y is

(a) [1, 4] (b) [2, 5] (c) [1, 5] (d) [2, 6]

142. If $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a} \perp \vec{b}, \vec{c}$ is inclined at the same angle to both $\vec{a}$ and $\vec{b}$ and $\vec{c} = p\vec{a} + q\vec{b} + r(\vec{a} \times \vec{b})$, then which of the following is true?

(a) $p = q$ (b) $|p| \le 1$
(c) $|q| \le 1$ (d) All of these

143. Area of the region bounded by the curves, $y = e^x$, $y = e^{-x}$ and the straight line $x = 1$ is given by
(a) $(e - e^{-1} + 2)$ sq. units
(b) $(e - e^{-1} - 2)$ sq. units
(c) $(e + e^{-1} - 2)$ sq. units
(d) none of these

144. If the normal at the end of latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through $(0, -b)$, then $e^4 + e^2$ (where e is eccentricity) equals
(a) 1 (b) $\sqrt{2}$
(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}+1}{2}$

145. A variable plane passes through the fixed point (a, b, c) and meets the axes at A, B, C. The locus of the point of intersection of the planes through A, B, C and parallel to the coordinate planes is
(a) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$ (b) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$
(c) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=-2$ (d) $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=-1$

146. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit coplanar vectors, then the scalar triple product $[2\vec{a}-\vec{b}\ \ 2\vec{b}-\vec{c}\ \ 2\vec{c}-\vec{a}]$ is equal to
(a) 0 (b) 1 (c) $-\sqrt{3}$ (d) $\sqrt{3}$

147. The total number of numbers that can be formed by using all the digits 1, 2, 3, 4, 3, 2, 1, so that the odd digits always occupy the odd places, is
(a) 3 (b) 6 (c) 9 (d) 18

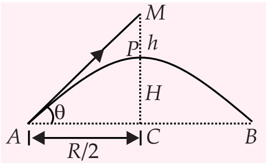
148. Number of real roots of the equation $\sqrt{x}+\sqrt{x-\sqrt{(1-x)}} = 1$ is
(a) 0 (b) 1 (c) 2 (d) 3

149. $\int \frac{xe^x}{(1+x)^2}dx$ is equal to
(a) $\frac{e^x}{x+1}+c$ (b) $e^x(x+1)+c$
(c) $-\frac{e^x}{(x+1)^2}+c$ (d) $\frac{e^x}{1+x^2}+c$

150. If $y=\sqrt{x+\sqrt{y+\sqrt{x+\sqrt{y+\ldots\infty}}}}$, then $\frac{dy}{dx}$ is equal to
(a) $\frac{1}{2y-1}$ (b) $\frac{y^2-x}{2y^3-2xy-1}$
(c) $2y - 1$ (d) none of these

1. **(a)** **2.** **(b)**

3. **(a) :**



From figure, $AC = R/2$, $PC = H$, $h = MP$ = ?

Given, $R = H$ or $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

or $\tan\theta = 4$

In ΔAMC, $\tan\theta = \frac{MC}{AC} = \frac{MP + PC}{AC}$

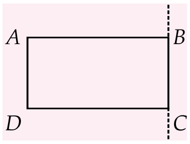
$\therefore \quad 4 = \frac{h+H}{R/2} \Rightarrow 4 = \frac{2(h+H)}{H} \Rightarrow h = H$

4. **(b) :** $\frac{[\text{Angular momentum}]}{[\text{Magnetic moment}]} = \left[\frac{2m}{q}\right]$

$= \left[\frac{2m}{It}\right] = \left[\frac{\text{M}}{\text{AT}}\right] = [\text{MA}^{-1}\ \text{T}^{-1}]$

5. **(d) :** $\frac{AB}{BC} = 2 \quad \therefore \quad AB = DC = \frac{l}{3}$

and $\quad BC = AD = \frac{l}{6}$



Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$

and $\quad m_{BC} = m_{AD} = \frac{m}{6}$

Moment of inertia about the desired axis is

$I = I_{AB} + I_{AD} + I_{DC} + I_{BC}$

$$= \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 + \left(\frac{m}{6}\right)\left(\frac{l}{3}\right)^2 + \frac{1}{3}\left(\frac{m}{3}\right)\left(\frac{l}{3}\right)^2 + 0 = \frac{7}{162}ml^2$$

6. **(d) :** Rotational kinetic energy is

$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}MK^2\left(\frac{v}{R}\right)^2$ ($\because$ $I = MK^2$ and $v = R\omega$)

$= \frac{1}{2}Mv^2\left(\frac{K^2}{R^2}\right)$

Translational kinetic energy, $K_T = \frac{1}{2}Mv^2$

As per question, $K_R = 40\%\ K_T$

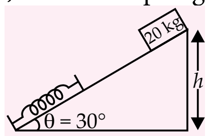
$\therefore \quad \frac{1}{2}Mv^2\left(\frac{K^2}{R^2}\right) = 40\%\frac{1}{2}Mv^2 \Rightarrow \frac{K^2}{R^2} = \frac{2}{5}$

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

Hence, the body is a solid ball.

7. **(b)**

8. **(c) :** As the spring is compressed by 2 m with the application of a force of 200 N, hence its spring constant k is given by



$k = \frac{F}{2} = \frac{200\ \text{N}}{2\ \text{m}} = 100\ \text{N m}^{-1}$

Suppose l be the distance along the inclined plane. Applying the conservation of energy,

$\frac{1}{2}kx_1^2 = mgh = mgl\sin\theta$

or $\frac{1}{2} \times 100 \times 4^2 = 20 \times 9.8 \times l \times \frac{1}{2}$

$\Rightarrow \quad l = \frac{800}{98} = 8.17\ \text{m}$

9. **(d)** **10.** **(d)**

11. **(b) :** According to law of conservation of mechanical energy

$\frac{1}{2}mu^2 - \frac{GMm}{R} = 0 \Rightarrow u^2 = \frac{2GM}{R}$

$u = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \qquad \left(\because g = \frac{GM}{R^2}\right)$

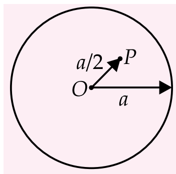
12. **(a)**

13. **(c) :** Here, mass of a particle = M

Mass of a spherical shell = M

Radius of a spherical shell = a

Gravitational potential at point P due to Q particle at O,



$V_1 = -\frac{GM}{a/2}$

Gravitational potential at point P due to spherical shell, $V_2 = -\frac{GM}{a}$

Hence, total gravitational potential at point P is

$V = V_1 + V_2 = -\frac{GM}{a/2} + \left(\frac{-GM}{a}\right) = -\frac{3GM}{a}$

$\because \quad |V| = \frac{3GM}{a}$

14. **(d)**

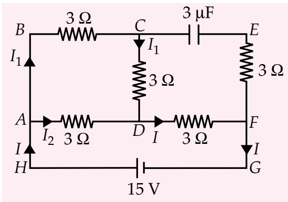
15. **(d) :** Here, $n_1 = 4$, $T_1 = 400$ K, $n_2 = 2$, $T_2 = 700$ K

The temperature of the mixture is

$T_{\text{mixture}} = \frac{n_1T_1 + n_2T_2}{n_1 + n_2} = \frac{1600\ \text{K} + 1400\ \text{K}}{6} = 500\ \text{K}$

16. **(b)**

17. **(c) :** When capacitor is fully charged, it draws no current. Hence potential difference across capacitor = Potential difference across C and F. Effective resistance of the network between A and D is



$$R_1 = \frac{(3+3)\times 3}{(3+3)+3} = 2\ \Omega$$

Total resistance of the circuit

= 2 Ω + 3 Ω = 5 Ω

Current, $I = \dfrac{15\text{ V}}{5\,\Omega}$ = 3 A

$$\therefore I_1 = \frac{(3\text{ A})(3\,\Omega)}{3\,\Omega + 6\,\Omega} = 1\text{ A and } I_2 = \frac{(3\text{ A})(6\,\Omega)}{3\,\Omega + 6\,\Omega} = 2\text{ A}$$

Potential difference across A and D = 3Ω × 2A = 6 V

Potential difference across D and F,

$V_D - V_F$ = 3 Ω × 3 A = 9 V

Potential difference across C and D,

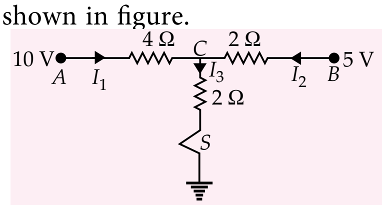
$V_C - V_D$ = 3 Ω × 1 A = 3 V

Potential difference across capacitor = $V_C - V_F$

= $(V_C - V_D) + (V_D - V_F)$ = 3 V + 9 V = 12 V

18. **(a)**

19. **(c) :** The currents through various arms will be as shown in figure.



Let V be the potential at C.

Applying Kirchhoff's first law at C, we get

$I_1 + I_2 = I_3$

$$\frac{10 - V}{4} + \frac{5 - V}{2} = \frac{V - 0}{2} \Rightarrow V = 4\text{ V}$$

$$\therefore \quad I_3 = \frac{4\text{ V}}{2\ \Omega} = 2\text{ A}$$

20. **(d) :** $\because\ 260 = \dfrac{1}{2l}\sqrt{\dfrac{T_1}{\mu}} \Rightarrow T_1 = 50.7g = 507\text{ N}$

When the mass is submerged, upthrust

= (0.0075 m^3)(10^3 kg m^{-3}) (10 m s^{-2}) = 75 N

New tension, T_2 = (507 – 75) N = 432 N

New fundamental frequency,

$$\upsilon = \frac{1}{2l}\sqrt{\frac{T_2}{\mu}}$$

$$\therefore \quad \frac{\upsilon}{260} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{432}{507}} = \frac{12}{13} \quad \text{or} \quad \upsilon = 240\ \text{Hz}$$

21. **(d) :** From the given graph, k = Slope of F-x graph

$$= \frac{(80-0)\text{ N}}{(0.2-0)\text{ m}} = 400\text{ N m}^{-1}$$

Time period of SHM is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.01\text{ kg}}{400\text{ N m}^{-1}}} = 0.03\text{ s}$$

22. **(a)**

23. **(b) :** The given equation of the transverse wave is $y = A\sin 2(\omega t - kx)$

Velocity of the particle = $\dfrac{dy}{dt}$ = $2A\omega\cos 2(\omega t - kx)$

Maximum velocity = $2A\omega$

$$\text{Velocity of the wave} = \frac{\text{Coefficient of } t}{\text{Coefficient of } x} = \frac{2\omega}{2k} = \frac{\omega}{k}$$

As per question,

$$2A\omega = \frac{\omega}{k} \text{ or } 2A = \frac{1}{k} = \frac{\lambda}{2\pi} \Rightarrow A = \frac{\lambda}{4\pi}$$

24. **(a)** **25.** **(c)**

26. **(b) :** A voltmeter is a galvanometer having a high resistance connected in series with it. The current through the galvanometer is

$$I_g = \frac{5\text{ V}}{300\ \Omega} = \frac{1}{60}\text{ A}$$

An ammeter is a galvanometer having a low resistance connected in parallel with it. The shunt resistance S is determined from

$$\frac{I_g}{I - I_g} = \frac{S}{G}$$

Given G = 300 Ω, I = 5 A, we get

$$\frac{1/60}{5-(1/60)} = \frac{S}{300} \Rightarrow S = 1\ \Omega$$

27. **(d)**

28. **(b) :** Magnetic flux linked with the loop is

$\phi = BA\cos\theta = B(\pi r^2)\cos 0° = B\pi r^2$

The magnitude of the induced emf is

$$|\varepsilon| = \frac{d\phi}{dt} = \frac{d}{dt}\left(B\pi r^2\right) = B\pi 2r\frac{dr}{dt}$$

= 0.025 × π × 2 × 2 × 10^{-2} × 1 × 10^{-3}

= π × 10^{-6} V = π μV

29. **(d) :** Here, $i = 5\sin\left(100t - \frac{\pi}{2}\right)$ A

$V = 200\sin(100t)$ V

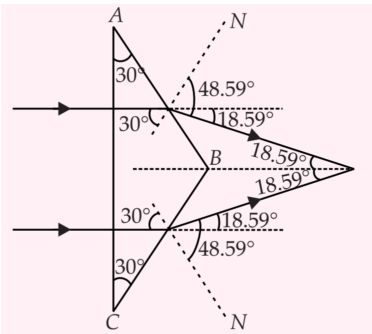
$\therefore$ Phase difference between V and i is, $\phi = \frac{\pi}{2}$

Power consumed, $P = V_{rms}\, i_{rms} \cos\phi$

$= V_{rms}\, i_{rms} \cos 90° =$ zero

30. **(b)** **31.** **(c)** **32.** **(b)**

33. **(b) :**



At the first surface AC, the ray will pass undeviated. Applying the Snell's law at second surface AB, we get

$\mu\sin 30° = 1\sin r \Rightarrow 1.5 \times \frac{1}{2} = \sin r$

$\sin r = 0.75 \Rightarrow r = 48.59$

$\therefore$ The angle between the incident horizontal ray and the emergent ray is 18.59°.

$\therefore$ The angle between the two emergent rays is nearly 37° *i.e.* 2 × 18.59°.

34. **(b) :** $\sigma = e[n_e\mu_e + n_h\mu_h]$

$= 1.6 \times 10^{-19}[5 \times 10^{18} \times 2.3 + 8 \times 10^{19} \times 0.01]$

$= 1.968\ \Omega^{-1}\ m^{-1}$

35. **(d)**

36. **(b) :** As $N = N_0 e^{-\lambda t}$ or $\frac{N}{N_0} = e^{-\lambda t}$

Taking natural logarithm on both sides, we get

$$-\lambda t = \ln\frac{N}{N_0} \text{ or } \lambda t = \ln\frac{N_0}{N} \text{ or } t = \frac{1}{\lambda}\ln\frac{N_0}{N}$$

$$\text{Here, } \lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{3.8} = 0.182, \frac{N}{N_0} = \frac{1}{20}$$

$$\therefore \quad t = \frac{1}{0.182}\ln(20) = 16.5 \text{ days}$$

37. **(a) :** In second excited state, $n = 3$,

So, $l_H = l_{Li} = 3\left(\frac{h}{2\pi}\right)$

while $E \propto Z^2$ and $Z_H = 1$, $Z_{Li} = 3$

So, $|E_{Li}| = 9|E_H|$ or $|E_H| < |E_{Li}|$

38. **(b) :** de Broglie wavelength associated with charged particle is given by,

$$\text{or } \lambda = \frac{h}{\sqrt{2mqV}}, \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}}$$

$$\text{As } \frac{m_p}{m_\alpha} = \frac{1}{4} \text{ and } \frac{q_\alpha}{q_p} = \frac{2}{1} \Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4\times 2} = 2\sqrt{2}$$

39. **(d) :** Efficiency of a Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

Here, $\eta = 25\%$, $T_2 = 27°C = 300$ K

$$\therefore \quad \frac{25}{100} = 1 - \frac{300}{T_1} \Rightarrow T_1 = 400 \text{ K} = 127°C$$

40. **(d)**

41. **(d) :** Alkaline earth metals are Group 2 elements. Emission of α-particle (^4_2He) will reduce its atomic number by 2 units and thus, displaces the daughter nuclei two positions left in the periodic table, thus to group 18, 16, 14, 12, 10, 8, 6 or 4, etc. As the last stable daughter nuclei formed could be either Pb (Group 14) or Bi (Group 15) therefore, the daughter nuclei would belong to Group 14.

42. **(a) :** Structures I and II differ only in the position of OH at C–1 and hence are anomers.

43. **(c)**

44. **(d) :** $\lambda = \frac{h}{\sqrt{2m(KE)}}$

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{m_{Ne}(KE)_{Ne}}{m_{He}(KE)_{He}}} \quad \ldots(i)$$

$$\Rightarrow \frac{m_{Ne}}{m_{He}} = \frac{20}{4} = 5 \quad \ldots(ii)$$

$KE \propto T$

$$\frac{(KE)_{Ne}}{(KE)_{He}} = \frac{727+273}{-73+273} = \frac{1000}{200} = 5 \quad \ldots(iii)$$

Put values from eqns. (ii) and (iii) in eqn. (i)

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{5\times 5} = 5$$

45. **(b) :** For transformations $L \to M$ and $N \to K$, volume is constant.

46. **(c) :** Mass of an organic compound = 1.4 g

$$\% \text{ of N} = \frac{1.4 \times \text{Meq. of acid consumed}}{\text{Mass of compound taken}}$$

Meq. of acid consumed

$$= \left(60 \times \frac{1}{10} \times 2\right) - \left(20 \times \frac{1}{10} \times 1\right) = 10$$
[Basicity of acid = 2]

$$\% \text{ of N} = \frac{1.4 \times 10}{1.4} = 10\%$$

47. **(c)** **48.** **(d)**

49. **(d) :** For '*n*' moles of gas, van der Waals' equation is :

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

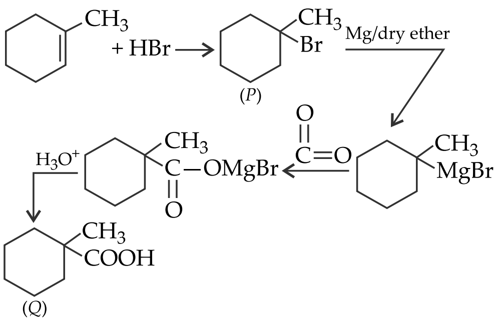
For 0.2 mol, $\left(P + \dfrac{0.04a}{V^2}\right)(V - 0.2b) = 0.2RT$

50. **(c) :** $\frac{1}{2}mv^2 = h(\upsilon - \upsilon_0) = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$

$$= hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = hc\left(\frac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$$

or $v^2 = \dfrac{2hc}{m}\left(\dfrac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)$; or $v = \left[\dfrac{2hc}{m}\left(\dfrac{\lambda_0 - \lambda}{\lambda_0 \lambda}\right)\right]^{1/2}$

51. **(d) :**



52. **(d) :** N is attached to four carbon atoms.

53. **(d) :** When the same quantity of electricity is passed through the solutions of different electrolytes, the weights of the elements deposited are directly proportional to their chemical equivalents.

i.e., $\dfrac{\text{Wt. of Cu deposited}}{\text{Wt. of Ag deposited}} = \dfrac{\text{Eq. wt. of Cu}}{\text{Eq. wt. of Ag}}$

$\Rightarrow \dfrac{W_{Cu}}{27} = \dfrac{63.6/2}{108/1}$ $\therefore$ $W_{Cu} = \dfrac{63.6 \times 27}{2 \times 108} = 7.95 \text{ g}$

54. **(c) :** Sum of oxidation numbers of all the atoms in $X_3(YZ_4)_3$ is zero *i.e.*
$3 \times (+3) + 3\,[1 \times (+5) + 4 \times (-2)] = 9 - 9 = 0$
In all other compounds, the sum of oxidation numbers is a finite number.

55. **(a)**

56. **(b) :** Mol. wt. of compound $= \dfrac{0.22 \times 22400}{112} = 44 \text{ g}$

$\% \text{ of C} = \dfrac{12}{44} \times \dfrac{0.44 \times 100}{0.22} = 54.54$

Amount of C in compound $= \dfrac{44 \times 54.54}{100} = 24$ g

$\therefore$ Molecular formula is C_2H_yO.
Now, corresponding to mol. wt. 44, molecular formula will be C_2H_4O.
Hence $x : y$ is 1 : 2.

57. **(c)**

58. **(b) :** Lower members, only, are soluble in water.

59. **(d) :** $A + 2B \rightleftharpoons C + D$;
x = mol of A remaining at equilibrium
Moles of A reacted = $1 - x$;
Moles of B reacted = $2(1 - x)$
Moles of B remaining = $3 - 2(1 - x) = 1 + 2x \approx 1$
($\because$ K is very large, $x << 1$)
Moles of C = Moles of D = $1 - x \approx 1$

Hence, $K = 1.0 \times 10^8 = \dfrac{[C][D]}{[A][B]^2} = \dfrac{1 \times 1}{x \times (1)^2} = \dfrac{1}{x}$

$\Rightarrow x = 1.0 \times 10^{-8}$ mol L^{-1}

60. **(b) :** $CH_3-\overset{\overset{NH}{||}}{C}-NH_2 + H^+ \longrightarrow CH_3-\overset{\overset{NH_2}{|}}{\overset{+}{C}}-NH_2$

$$\updownarrow$$

$CH_3-\overset{\overset{NH_2}{|}}{C}=\overset{+}{N}H_2 \longleftrightarrow CH_3-\overset{\overset{\overset{+}{N}H_2}{||}}{C}-NH_2$

The conjugate acid obtained by addition of a proton to (I) is stabilized by two equivalent resonance structures and hence compound (I) is the most basic. Further 2° amines are more basic than 1° amines while amides are least basic due to delocalization of lone pair of electrons of N over the CO group. Thus the order is
I > III > II > IV.

61. **(c) :** It is a hydroboration-oxidation reaction. It is the addition of H_2O according to anti-Markownikoff's rule. Hence, terminal carbon gets the – OH group.

62. **(c)**

63. **(d) :** $H_3\overset{+}{N}\overset{\overset{R}{|}}{C}HCOOH$ (Low pH) $\underset{H^+}{\overset{OH^-}{\rightleftharpoons}}$ $H_3\overset{+}{N}\overset{\overset{R}{|}}{C}HCOO^-$ (Intermediate pH)

$$\Big\Updownarrow \; H^+ / OH^-$$

$H_2N\overset{\overset{R}{|}}{C}HCOO^-$ (High pH)

$$K_1 = \frac{[H_3\overset{+}{N}CHRCOO^-][H^+]}{[H_3\overset{+}{N}CHRCOOH]}$$

$$K_2 = \frac{[H_2NCHRCOO^-][H^+]}{[H_3\overset{+}{N}CHRCOO^-]}$$

Thus, $K_1K_2 = \dfrac{[H_2NCHRCOO^-][H^+]^2}{[H_3\overset{+}{N}CHRCOOH]}$

At the isoelectric point,

$[H_2NCHRCOO^-] = [H_3N^+CHRCOOH]$

$K_1K_2 = [H^+]^2$; $2\log[H^+] = \log K_1 + \log K_2$

$-2\log[H^+] = -\log K_1 - \log K_2$

$2pH = pK_1 + pK_2$

or $pH = (pK_1 + pK_2)/2$

64. **(c) :** $\dfrac{-d[A]}{dt} = k[A]^{1/3} \Rightarrow -\int \dfrac{d[A]}{[A]^{1/3}} = \int k\,dt$

$$\Rightarrow -\frac{3}{2}[A]^{2/3} = kt + C \quad \ldots\text{(i)}$$

At $t = 0$; $[A] = [A_0]$

Then, $-\dfrac{3}{2}[A_0]^{2/3} = C$

Putting the value of C in eq. (i), we get

$$-\frac{3}{2}[A]^{2/3} = kt - \frac{3}{2}[A_0]^{2/3}$$

$$\Rightarrow kt = \frac{3}{2}[A_0]^{2/3} - \frac{3}{2}[A]^{2/3}$$

At $t = t_{1/2}$; $[A] = \dfrac{[A_0]}{2}$

$$\therefore kt_{1/2} = \frac{3}{2}[A_0]^{2/3} - \frac{3}{2}\left[\frac{A_0}{2}\right]^{2/3}$$

$$\Rightarrow kt_{1/2} = \frac{3}{2}\left[[A_0]^{2/3} - \frac{[A_0]^{2/3}}{(2)^{2/3}}\right]$$

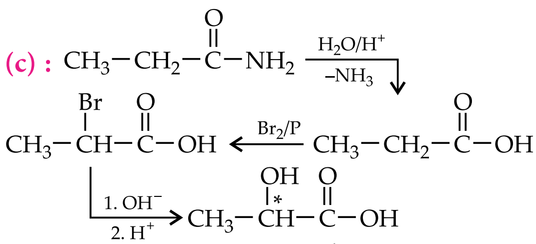
$$\Rightarrow t_{1/2} = \frac{3}{2k}[A_0]^{2/3}\left(1 - \frac{1}{(2)^{2/3}}\right)$$

$$\Rightarrow t_{1/2} = \frac{3[A_0]^{2/3}}{2k}\left[\frac{2^{2/3}-1}{2^{2/3}}\right]$$

$$\therefore t_{1/2} = \frac{3[A_0]^{2/3}}{k}\left[\frac{2^{2/3}-1}{2^{5/3}}\right]$$

65. **(c)** **66.** **(b)**

67. **(c) :**

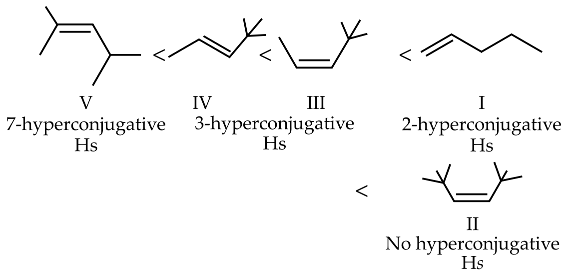


Lactic acid

68. **(c) :** As the volume is decreased, pressure increases because more number of molecules strike the surface per second.

69. **(c) :** Greater is the stability of alkene, lower the heat of hydrogenation.

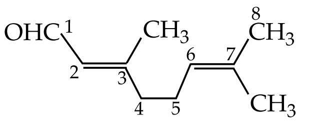
Out of *cis* and *trans* isomers, *trans* isomer is more stable than *cis* isomer in which two alkyl groups lie on the same side of the double bond and hence cause steric hindrance, therefore, heat of hydrogenation of *trans* isomer is *less* than that of *cis* isomer.



70. **(a)**

71. **(d) :** Octahedral complexes of type $[MA_5B]$ does not show geometrical isomerism.

72. **(d) :**



3,7-Dimethylocta-2,6-dien-1-al

73. **(b) :** As we move down the group, the atomic size increases and electronegativity decreases, and, thereby, tendency to show catenation decreases. The order of catenation is C >> Si > Ge ≈ Sn. Lead does not show catenation.

74. **(b) :** Suppose number of O^{2-} ions = n. Then number of octahedral voids = n and number of tetrahedral voids = $2n$

No. of cations X present in tetrahedral voids

$$= \frac{1}{6} \times 2n = \frac{n}{3}$$

No. of cations Y present in octahedral voids

$$= \frac{1}{3} \times n = \frac{n}{3}$$

$\therefore$ Ratio $X : Y : O^{2-} = \frac{n}{3} : \frac{n}{3} : n = 1 : 1 : 3$

Hence, formula is XYO_3.

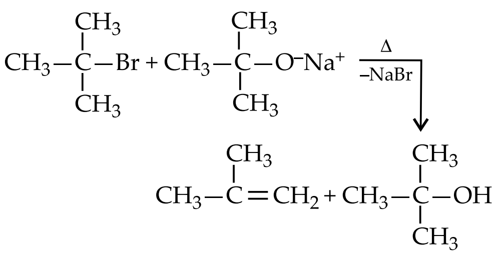
75. (c) : $\Delta v = \frac{0.005}{100} \times 600 \text{ m s}^{-1} = 3 \times 10^{-2} \text{ m s}^{-1}$

$$\Delta x \times m\Delta v = \frac{h}{4\pi}; \quad \therefore \ \Delta x = \frac{h}{4\pi m \Delta v}$$

$$\Delta x = \frac{6.6 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}}{4 \times 3.14 \times 9.1 \times 10^{-31} \text{kg} \times 3 \times 10^{-2} \text{m s}^{-1}}$$

$= 1.92 \times 10^{-3}$ m

76. (d) : Di-*tert*.-butyl ether cannot be prepared by Williamson's synthesis, since *tert*.-alkyl halides prefer to undergo elimination rather than substitution, *i.e.*,



77. (a) : 1° alky halides (*i.e.*, CH_3Br) undergo S_N2 reaction.

78. (b) : The low solubility of LiF in water is due to its high lattice enthalpy. Smaller Li^+ ion is stabilised by smaller F^- ion.

79. (c) : The smaller the size of the cation and the larger the size of the anion, the greater is the covalent character of an ionic bond.

80. (a)	**81. (b)**	**82. (b)**	**83. (c)**	**84. (c)**
85. (d)	**86. (c)**	**87. (a)**	**88. (b)**	**89. (c)**
90. (c)	**91. (d)**	**92. (b)**	**93. (d)**	**94 (c)**
95. (c)	**96. (b)**	**97. (b)**	**98. (c)**	**99. (a)**
100. (b)	**101. (b)**	**102. (c)**	**103. (a)**	**104. (b)**
105. (c)				

106. (b) : $\because$ Numbers a, b, 12 are in G.P.

$\therefore$ $b^2 = 12a$ ...(i)

and a, b, 9 are in A.P.

$\therefore$ $2b = a + 9$ or $a = 2b - 9$ ...(ii)

From Eqs. (i) and (ii), we get

$b^2 = 12(2b - 9) \Rightarrow b^2 - 24b + 108 = 0$

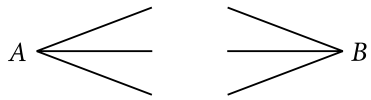
$(b - 18)(b - 6) = 0 \quad \therefore \quad b = 6, 18$

From Eq. (ii), $a = 3, 27$ respectively.

$\therefore$ Common ratio $= \frac{b}{a} = \frac{6}{3}$ and $\frac{18}{27} = 2$ and $\frac{2}{3}$

$\therefore$ Common ratio $= \frac{2}{3}$ (for decreasing G.P. common ratio $\neq 2$)

107. (a) :



13 pass through A 11 pass through B

$\therefore$ Number of intersection points

$= {}^{37}C_2 - {}^{13}C_2 - {}^{11}C_2 + 2 = 535$

($\because$ Two points A and B)

108. (b)

109. (b) : The probability of hitting in one shot

$= \frac{10}{100} = \frac{1}{10}$

If he fires n shots, the probability of hitting atleast once

$$= 1 - \left(1 - \frac{1}{10}\right)^n = 1 - \left(\frac{9}{10}\right)^n = 50\% = \frac{50}{100} = \frac{1}{2}$$

$$\Rightarrow \left(\frac{9}{10}\right)^n = \left(\frac{1}{2}\right)$$

$\therefore$ $n\{\log 9 - \log 10\} = \log 1 - \log 2$

$\Rightarrow$ $n\{2 \log 3 - 1\} = 0 - \log 2$

$$\therefore \ n = \frac{\log 2}{1 - 2\log 3} = \frac{0.3010300}{1 - 2 \times 0.4771213} = 6.6$$

Hence, $n = 7$

110. (c)

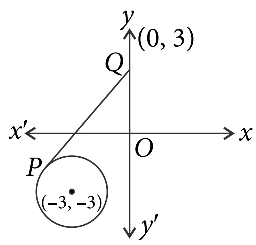
111. (c) : $\because$ Q lies on y-axis,

Put $x = 0$

in $5x - 2y + 6 = 0$

$\therefore$ $y = 3$

$\Rightarrow$ $Q \equiv (0, 3)$



$\therefore$ $PQ = \sqrt{0^2 + 3^2 + 0 + 6 \times 3 - 2} = \sqrt{25} = 5$

112. (c) : Since, $A \equiv z_1$, $B \equiv z_2$, $C \equiv (1 - i)z_1 + iz_2$

$\therefore$ $AB = |z_1 - z_2|$,

$BC = |(1 - i)z_1 + iz_2 - z_2| = |1 - i|\,|z_1 - z_2|$

$= \sqrt{2}\,|z_1 - z_2|$

and $CA = |(1 - i)z_1 + iz_2 - z_1|$

$= |i|\,|-z_1 + z_2| = |z_1 - z_2|$

$\therefore$ $AB = CA$ and $(AB)^2 + (CA)^2 = (BC)^2$.

113. (c) : Since, $y = |x - 1| = \begin{cases} x - 1, & x \geq 1 \\ 1 - x, & x < 1 \end{cases}$ ...(i)

and $y = 3 - |x| = \begin{cases} 3-x, & x \ge 0 \\ 3+x, & x < 0 \end{cases}$ ...(ii)

Solving eqs. (i) and (ii), we get

$x = 2$ and $x = -1$

$\therefore$ Required area $= \left|\int_{-1}^{2}(3-|x|-|x-1|)dx\right|$

$= \left|\int_{-1}^{0}(2x+2)dx + \int_{0}^{1}2dx + \int_{1}^{2}(4-2x)dx\right|$

= 4 sq. units.

114. (d) : D.R.'s of PQ are $1 - (-5), 5 - 3, -2 - 1$

i.e., 6, 2, –3

and $\sqrt{6^2+2^2+(-3)^2} = 7$ $\therefore$ D.C.'s are $\frac{6}{7}, \frac{2}{7}, \frac{3}{-7}$

115. (c) : $(1 + \cos \pi/8)(1 + \cos 3\pi/8)(1 + \cos 5\pi/8)(1 + \cos 7\pi/8)$

$= 2\cos^2(\pi/16) \cdot 2\cos^2(3\pi/16) \cdot 2\cos^2(5\pi/16) \cdot 2\cos^2(7\pi/16)$

$= 16[\cos(\pi/16)\cos(3\pi/16)\cos(5\pi/16)\cos(7\pi/16)]^2$

$= [2\cos(7\pi/16)\cos(\pi/16)]^2\,[2\cos(5\pi/16)\cos(3\pi/16)]^2$

$= [\cos(\pi/2) + \cos(3\pi/8)]^2\,[\cos(\pi/2) + \cos(\pi/8)]^2$

$= \cos^2(3\pi/8)\cos^2(\pi/8)$

$= \frac{1}{4}(\cos \pi/2 + \cos \pi/4)^2 = \frac{1}{8}$

116. (d) : For parallel, $\vec{a} \times \vec{b} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 4 & -\lambda & 6 \end{vmatrix} = 0$$

$\Rightarrow \hat{i}(6+3\lambda) - \hat{j}(0) + \hat{k}(-2\lambda - 4) = 0$

$\therefore 6 + 3\lambda = 0 \Rightarrow \lambda = -2$

117. (b) : We have,

$$f(x) = \begin{cases} \dfrac{a|x^2-x-2|}{2+x-x^2}, & x<2 \\ b, & x=2 \\ \dfrac{x-[x]}{x-2}, & x>2 \end{cases}$$

$$f(x) = \begin{cases} -\dfrac{a(x^2-x-2)}{2+x-x^2}, & x<2 \\ b, & x=2 \\ \dfrac{x-[x]}{x-2}, & x>2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} a, & x<2 \\ b, & x=2 \\ \dfrac{x-[x]}{x-2}, & x>2 \end{cases}$$

Now,

L.H.L. $= \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h) = \lim_{h\to 0} a = a$

R.H.L. $= \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)$

$= \lim_{h\to 0} \frac{2+h-[2+h]}{2+h-2} = \lim_{h\to 0} \frac{2+h-2}{2+h-2} = 1$

Value of the function $= f(2) = b$. Since $f(x)$ is continuous at $x = 2$

$\therefore$ L.H.L. = R.H.L. = Value of $f(x)$

$\therefore a = b = 1$

118. (d) : Let $f(x) = a^2x^2 + 2bx + 2c$

From the question, $a^2\alpha^2 + b\alpha + c = 0$ and $a^2\beta^2 - b\beta - c = 0$

Now, $f(\alpha) = a^2\alpha^2 + 2b\alpha + 2c = b\alpha + c = -a^2\alpha^2$

$f(\beta) = a^2\beta^2 + 2b\beta + 2c = 3(b\beta + c) = 3a^2\beta^2$

But $0 < \alpha < \beta \Rightarrow \alpha, \beta$ are real

$\therefore f(\alpha) < 0, f(\beta) > 0$

Hence, $\alpha < \gamma < \beta$.

119. (d) : In the given expansion, $(r + 1)^{\text{th}}$ term $= T_{r+1}$

$= {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\sqrt{\frac{3}{2x^2}}\right)^{r}$

$= {}^{10}C_r\left(\frac{x}{3}\right)^{5-\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{r}{2}} = {}^{10}C_r\frac{x^{5-\frac{r}{2}-r}}{3^{5-\frac{r}{2}-\frac{r}{2}}\cdot 2^{\frac{r}{2}}}$

For independent of x,

Put $5 - \frac{r}{2} - r = 0$

$\therefore 5 = \frac{3r}{2} \Rightarrow r = \frac{10}{3}$, impossible

$\because r \ne$ whole number

120. (b) **121. (c)**

122. (d) : Let each line $\ell \in$ set of the lines (L)

(i) As $\ell \parallel \ell \Rightarrow (\ell, \ell) \in R\ \forall\ \ell \in L$

$\Rightarrow R$ is reflexive.

(ii) Let $\ell_1, \ell_2 \in L$ such that $(\ell_1, \ell_2) \in R$ then

$\ell_1 \parallel \ell_2 \Rightarrow \ell_2 \parallel \ell_1 \Rightarrow (\ell_2, \ell_1) \in R$

$\therefore R$ is symmetric.

(iii) Let $\ell_1, \ell_2, \ell_3 \in L$ such that $(\ell_1, \ell_2) \in R$ and $(\ell_2, \ell_3) \in R$

$\Rightarrow \ell_1 \parallel \ell_2 \parallel \ell_3 \Rightarrow (\ell_1, \ell_3) \in R$

$\Rightarrow R$ is transitive.

Since a relation R, which is reflexive, symmetric and transitive is known as equivalence relation.

$\therefore$ Given relation is an equivalence relation.

123. (c) : $P(n) = 5^n - 2^n$

Let $n = 1 \Rightarrow P(1) = 3\lambda = 3$

$\therefore\ \lambda = 1$

Similarly $n = 5 \therefore P(5) = 5^5 - 2^5$

$= 3125 - 32 = 3093 = 3 \times 1031$

In this case, $\lambda = 1031$

Similarly, we can check the result for other cases and find that the least value of λ and n is 1.

124. (b) : Rank of diagonal matrix = Order of matrix = 3.

125. (b) : There are 11 letters in the word 'PROBABILITY' out of which 1 can be selected in ${}^{11}C_1$ ways.

$\therefore$ Exhaustive number of cases = ${}^{11}C_1 = 11$

There are three vowels viz AIO.

Therefore, favourable number of cases = ${}^{3}C_1 = 3$

Hence, the required probability = $\dfrac{3}{11}$

126. (c) : Total number of functions from $A \to B = 3^4 = 81$

Number of onto mappings = Coefficient of x^4 in $4!(e^x - 1)^3$.

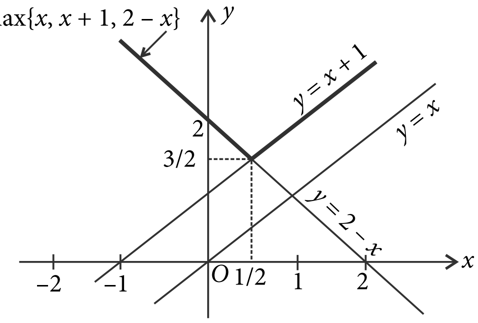
= Coefficient of x^4 in $4!(e^{3x} - 3e^{2x} + 3e^x - 1)$

$= 4!\left(\dfrac{3^4}{4!} - \dfrac{3\cdot 2^4}{4!} + \dfrac{3\cdot 1^4}{4!} - 0\right)$

$= 81 - 48 + 3 = 81 - 45 = 36$

$\therefore$ Number of functions from $A \to B$, which are not onto is $81 - 36 = 45$

127. (d) : $\because f(x) = \text{maximum } \{x, x + 1, 2 - x\}$



$\therefore$ Minimum value of function = 3/2

128. (d)

129. (c) : $A = \{x : x = 4n + 1\ \forall\ 2 \le n \le 6\}$

or $A = \{9, 13, 17, 21, 25\}$

$\Rightarrow$ Number of elements of $A = 5$

$\therefore$ Number of subsets of $A = 2^n = 2^5$

130. (c) : Since, $\bar{x}$ = Mean of the series

$= \dfrac{a + (a + d) + ... + (a + 2nd)}{2n + 1} = a + nd$

x_i	$d = \lvert x_i - \bar{x} \rvert$	$\lvert d \rvert^2 = D$
a	nd	n^2d^2
$a + d$	$(n - 1)d$	$(n - 1)^2d^2$
$\vdots$	$\vdots$	$\vdots$
$a + (n - 2)d$	$2d$	$4d^2$
$a + (n - 1)d$	d	d^2
$a + nd$	0	0
$a + (n + 1)d$	d	d^2
$a + (n + 2)d$	$2d$	$4d^2$
$\vdots$	$\vdots$	$\vdots$
$a + 2nd$	nd	n^2d^2
	$\Sigma\lvert d\rvert = 2dn\left(\dfrac{n+1}{2}\right)$	$\Sigma\lvert d\rvert^2 = 2d^2 [1^2 + 2^2 + ... + n^2]$

We have $\Sigma|d| = n(n + 1)\, d$ and

$\Sigma|d^2| = \dfrac{2d^2 (n)(n+1)(2n+1)}{6}$

Now, M.D. $= \dfrac{\Sigma|d|}{N}$ and $\sigma^2 = \dfrac{\Sigma|d|^2}{N}$

M.D. $= \dfrac{n(n+1)d}{2n+1}$ and $\sigma^2 = \dfrac{2d^2 \cdot n(n+1)(2n+1)}{6\cdot(2n+1)}$

$= \dfrac{n(n+1)d^2}{3}$

$\therefore$ S.D. $= \sqrt{\sigma^2} = \sqrt{\dfrac{n(n+1)}{3}}\cdot d$

131. (a) : Let $S_n = Pn^2 + Qn$ = Sum of first n terms

According to question,

Sum of first $3n$ terms = Sum of the next n terms

$\Rightarrow S_{3n} = S_{4n} - S_{3n}$ or $2S_{3n} = S_{4n}$

or $2[P(3n)^2 + Q(3n)] = P(4n)^2 + Q(4n)$

$\Rightarrow 2Pn^2 + 2Qn = 0$ or $Q = -nP$ ...(i)

Then,

$\dfrac{\text{Sum of first } 2n \text{ terms}}{\text{Sum of next } 2n \text{ terms}} = \dfrac{S_{2n}}{S_{4n} - S_{2n}}$

$= \dfrac{P(2n)^2 + Q(2n)}{[P(4n)^2 + Q(4n)] - [P(2n)^2 + Q(2n)]}$

$= \dfrac{2nP + Q}{6Pn + Q} = \dfrac{nP}{5nP} = \dfrac{1}{5}$ [From eq. (i)]

132. (b) : Let $P = \lim\limits_{n\to\infty} \dfrac{(n!)^{1/n}}{n} = \lim\limits_{n\to\infty}\left(\dfrac{n!}{n^n}\right)^{1/n}$

$$= \lim_{n\to\infty}\left(\frac{1\cdot 2\cdot 3\cdot 4.....n}{n\cdot n\cdot n......n}\right)^{1/n}$$

$$= \lim_{n\to\infty}\left(\left(\frac{1}{n}\right)\left(\frac{2}{n}\right)\left(\frac{3}{n}\right).....\left(\frac{n}{n}\right)\right)^{1/n}$$

$\therefore$ ln P =

$$\lim_{n\to\infty}\frac{1}{n}\left[\ln\left(\frac{1}{n}\right)+\ln\left(\frac{2}{n}\right)+\ln\left(\frac{3}{n}\right)+...+\ln\left(\frac{n}{n}\right)\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right) = \int_0^1 \ln x\, dx$$

$= [\ln x \cdot x - x]_0^1 = 0 - 1 = -1 \quad \therefore \quad P = e^{-1}$

133. (b)

134. (b) : If eccentricities of ellipse and hyperbola are e and e_1.

$\therefore$ Foci are $(\pm ae, 0)$ and $(\pm a_1 e_1, 0)$.

Here, $ae = a_1 e_1 \quad \Rightarrow \quad a^2e^2 = a_1^2 e_1^2$

$$a^2\left(1-\frac{b^2}{a^2}\right) = a_1^2\left(1+\frac{b_1^2}{a_1^2}\right)$$

$\Rightarrow a^2 - b^2 = a_1^2 + b_1^2 \quad \Rightarrow 25 - b^2 = \dfrac{144}{25}+\dfrac{81}{25} = 9$

$\therefore \quad b^2 = 16$

135. (b) : $\cos^{-1}x + \cos^{-1}y = \dfrac{\pi}{2} - \sin^{-1}x + \dfrac{\pi}{2} - \sin^{-1}y$

$= \pi - (\sin^{-1}x + \sin^{-1}y) = \pi - \dfrac{2\pi}{3} = \dfrac{\pi}{3}$ (given)

136. (b) : $\left(1+\dfrac{1}{1}\right)^n = 64 \quad \Rightarrow \quad 2^n = 2^6 \quad \therefore \quad n = 6$

General term $T_{r+1} = {}^nC_r(x)^{n-r}\left(\dfrac{1}{x}\right)^r = {}^6C_r\, x^{6-2r}$

For independent of x,

$6 - 2r = 0 \quad \Rightarrow \quad r = 3$

$\therefore \quad T_{3+1} = {}^6C_3 x^0 = {}^6C_3 = 20$

137. (b) : $\because \quad \sin y\dfrac{dy}{dx} = \cos y(1 - x\cos y)$

$\Rightarrow \tan y\dfrac{dy}{dx} = 1 - x\cos y$

$\Rightarrow \sec y\tan y\dfrac{dy}{dx} - \sec y = -x \quad$ ...(i)

Put $\sec y = v \quad \Rightarrow \quad \sec y\tan y\dfrac{dy}{dx} = \dfrac{dv}{dx}$

Then, from eq. (i), $\dfrac{dv}{dx} - v = -x$

$\therefore$ IF $= e^{\int -1\cdot dx} = e^{-x}$

Then, the solution is

$v\,.\,(e^{-x}) = \int(-x)e^{-x}dx$

$\Rightarrow v\cdot e^{-x} = (-x)(-e^{-x}) + e^{-x} + c$

or $v = x + 1 + ce^x$ or $\sec y = x + 1 + ce^x$

138. (c)

139. (c) : Out of 6 novels, 4 novels can be selected in ${}^6C_4$ ways.

Also out of 3 dictionaries, 1 dictionary can be selected in ${}^3C_1$ ways.

Since the dictionary is fixed in the middle, we only have to arrange 4 novels which can be done in 4! ways.

Then the number of ways $= {}^6C_4\cdot{}^3C_1\cdot 4!$

$$= \frac{6\cdot 5}{2}\cdot 3\cdot 24 = 1080$$

140. (a) : Normal at t, *i.e.* $(at^2, 2at)$ is

$tx + y = 2at + at^3$ or, $y = -tx + (2at + at^3)$

This will touch $x^2 - y^2 = a^2$ or $\dfrac{x^2}{a^2}-\dfrac{y^2}{a^2} = 1$ if

$(2at + at^3)^2 = a^2(-t)^2 - a^2$ [Using $c^2 = a^2m^2 - b^2$]

$\Rightarrow 4t^2 + t^6 + 4t^4 = t^2 - 1$

$\Rightarrow t^6 + 3t^4 + 3t^2 + 1 = -t^4 \Rightarrow (t^2 + 1)^3 = -t^4 < 0$

[Hence if $t = 0$ then normal to $y^2 = 4ax$ will be x-axis i.e. (0, 0) and x-axis can't touch rectangular hyperbola $x^2 - y^2 = a^2$ $\therefore t \neq 0$]

141. (d) : Rewrite $f(x) = 2\sin(x + \pi/6) + 4$

or $f(x) = 2\cos\left(x-\dfrac{\pi}{3}\right) + 4$

$\therefore \quad Y = [2, 6]$ ($\because$ min and max values of $\sin\theta$ and $\cos\theta$ are -1 and $+1$)

142. (d) : $\because \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

Let θ be the angle between $\vec{a}\,\&\,\vec{c}$ and $\vec{b}\,\&\,\vec{c}$

$\therefore \quad \cos\theta = \dfrac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|} = \vec{a}\cdot\vec{c} \quad$ ...(i)

Similarly, $\cos\theta = \vec{b}\cdot\vec{c} \quad$ ...(ii)

$\because \quad \vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0$

$\because \quad \vec{c} = p\vec{a} + q\vec{b} + r(\vec{a}\times\vec{b})$

$\therefore \quad \vec{a}\cdot\vec{c} = p\vec{a}\cdot\vec{a} + q\,\vec{a}\cdot\vec{b} + r\,\vec{a}\cdot(\vec{a}\times\vec{b})$

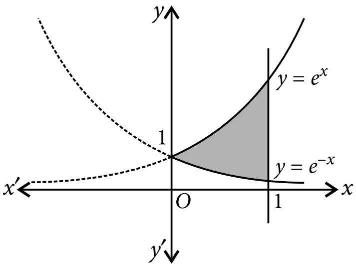
$= p + 0 + 0 = p \Rightarrow \cos\theta = p$ [from (i)]

$\therefore \quad |p| = |\cos\theta| \leq 1$

Similarly, $\cos\theta = q \Rightarrow |q| \leq 1$

Also, $p = q$

143. (c) :



$\therefore$ Required area $= \int_0^1 (e^x - e^{-x})dx$

$= [e^x + e^{-x}]_0^1 = (e + e^{-1}) - (e^0 + e^{-0})$
$= (e + e^{-1} - 2)$ sq. units.

144. (a) : Normal at the extremity of latus rectum in the first quadrant $(ae, b^2/a)$ is

$$\frac{x - ae}{ae/a^2} = \frac{y - b^2/a}{b^2/ab^2}$$

As it passes through $(0, -b)$

$$\frac{-ae}{ae/a^2} = \frac{-b - b^2/a}{1/a}$$

$\Rightarrow -a^2 = -ab - b^2 \Rightarrow a^2 - b^2 = ab$
$\Rightarrow a^2e^2 = ab$ or $e^2 = b/a$

$\therefore e^4 = \frac{b^2}{a^2} = 1 = e^2 \Rightarrow e^4 + e^2 = 1$

145. (b) : Let plane is $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$

which passes through (a, b, c).

$\therefore \frac{a}{x_1} + \frac{b}{y_1} + \frac{c}{z_1} = 1$

$\therefore$ Locus of (x_1, y_1, z_1) is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$

146. (a) : Given, $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

Also, $|\vec{a}| = 1$, $|\vec{b}| = 1$ and $|\vec{c}| = 1$

$\because [2\vec{a} - \vec{b}\ \ 2\vec{b} - \vec{c}\ \ 2\vec{c} - \vec{a}]$
$= (2\vec{a} - \vec{b}) \cdot \{(2\vec{b} - \vec{c}) \times (2\vec{c} - \vec{a})\}$
$= (2\vec{a} - \vec{b}) \cdot \{4\vec{b} \times \vec{c} - 2\vec{b} \times \vec{a} - 2\vec{c} \times \vec{c} + \vec{c} \times \vec{a}\}$
$= (2\vec{a} - \vec{b}) \cdot \{4(\vec{b} \times \vec{c}) + 2(\vec{a} \times \vec{b}) - 0 + (\vec{c} \times \vec{a})\}$
$= 8\vec{a} \cdot (\vec{b} \times \vec{c}) + 4\vec{a} \cdot (\vec{a} \times \vec{b}) + 2\vec{a} \cdot (\vec{c} \times \vec{a})$
$\quad - 4\vec{b} \cdot (\vec{b} \times \vec{c}) - 2\vec{b}(\vec{a} \times \vec{b}) - \vec{b}(\vec{c} \times \vec{a})$
$= 8[\vec{a}\ \vec{b}\ \vec{c}] + 0 + 0 - 0 - 0 - [\vec{a}\ \vec{b}\ \vec{c}]$
$= 7[\vec{a}\ \vec{b}\ \vec{c}] = 0 \qquad (\because [\vec{a}\ \vec{b}\ \vec{c}] = 0)$

147. (d) : Required numbers $= \frac{4!}{2!2!} \times \frac{3!}{2!} = 18$

148. (b) : Given $\sqrt{x} + \sqrt{x - \sqrt{(1-x)}} = 1$ ...(i)

$\Rightarrow \sqrt{x - \sqrt{(1-x)}} = 1 - \sqrt{x}$ ...(ii)

Squaring (ii) both sides, we get

$x - \sqrt{1-x} = 1 + x - 2\sqrt{x}$

$\Rightarrow -\sqrt{(1-x)} = 1 - 2\sqrt{x}$ ...(iii)

Squaring (iii) both sides, we get

$\Rightarrow 1 - x = 1 + 4x - 4\sqrt{x} \quad \Rightarrow \quad 4\sqrt{x} = 5x$

$\Rightarrow 25x^2 - 16x = 0 \quad \Rightarrow \quad x = 0, \frac{16}{25}$

$\Rightarrow x = \frac{16}{25} \qquad (\because x = 0$ does not satisfy eq. (i))

149. (a) : Let $I = \int \frac{xe^x}{(1+x)^2} dx$

$= \int e^x \left[\frac{(1+x) - 1}{(1+x)^2}\right] dx = \int e^x \left\{\frac{1}{1+x} - \frac{1}{(1+x)^2}\right\} dx$

$= \frac{e^x}{(1+x)} + c$
$\qquad (\because \int e^x \{(f(x) + f'(x)\} dx = e^x f(x) + c)$

150. (b) : We have, $y = \sqrt{x + \sqrt{y + y}}$

$\Rightarrow (y^2 - x) = \sqrt{2y} \quad \Rightarrow \quad (y^2 - x)^2 = 2y$

Differentiating both sides w.r.t. x, we get

$$2(y^2 - x)\left(2y\frac{dy}{dx} - 1\right) = 2\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(y^2 - x)}{2y^3 - 2xy - 1}$$

PRACTICE PAPER

JEE ADVANCED

SECTION-1

SINGLE CORRECT ANSWER TYPE

1. Suppose $\tan\alpha = \frac{p}{q}$ where p, q are integers and $q \neq 0$ and $\tan 3\alpha = \tan 2\beta$ such that $\tan\beta$ is rational, then $p^2 + q^2$ is a
(a) perfect square (b) perfect cube
(c) perfect 5^{th} power
(d) perfect 6^{th} power of an integer

2. $\sum_{k=0}^{n} {}^nC_k \left(\tan\frac{x}{2}\right)^{2k}\left[1+2^k\cdot\frac{1}{(1-\tan^2 x/2)^k}\right]=$
(a) $\sec^{2n}\left(\frac{x}{2}\right)+\sec^n x$
(b) $\operatorname{cosec}^{2n}\left(\frac{x}{2}\right)+\operatorname{cosec}^{2n} x$
(c) $\tan^{2n}\left(\frac{x}{2}\right)+\tan^n x$
(d) $\cot^{2n}\left(\frac{x}{2}\right)+\cot^n x$

3. The number of positive roots of the equation $x^x = \frac{1}{\sqrt{2}}$ is/are
(a) 0 (b) 1 (c) 2 (d) 4

4. Consider the equation $x^5 + 5\lambda x^4 - x^3 + (\lambda\alpha - 4)x^2 - (8\lambda + 3)x + \lambda\alpha - 2 = 0$. The value of α such that the given equation has exactly 2 roots independent of λ is
(a) -2 (b) -3 (c) 2 (d) 3

5. Let S be the set of all points (x, y) in the plane such that $x, y \in [0, \pi/2]$. The area of the subset of S for which $\sin^2 x + \sin^2 y - \sin x \sin y \leq 3/4$ is
(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi^2}{4}$ (c) $\frac{\pi^2}{5}$ (d) $\frac{\pi^2}{6}$

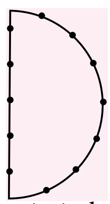
6. Let $ABCD$ be a tetrahedron. Let 'a' be the length of edge AB and let Δ be the area of projection of the tetrahedron on a plane, perpendicular to AB then volume of the tetrahedron is
(a) $\frac{a\Delta}{2}$ (b) $\frac{a\Delta}{3}$ (c) $\frac{a\Delta}{4}$ (d) $\frac{a\Delta}{5}$

7. Let z_1, z_2, z_3 be distinct complex numbers such that $|z_1| = |z_2| = |z_3| = r$, $z_2 \neq z_3$, $a \in R$, then minimum value of $|az_2 + (1 - a)z_3 - z_1|$ is
(a) $\frac{|z_1-z_2||z_1-z_3|}{r}$ (b) $\frac{|z_1-z_2||z_1-z_3|}{2r}$
(c) $\frac{|z_1-z_2||z_2-z_3|}{r}$ (d) $\frac{|z_1-z_2||z_2-z_3|}{2r}$

8. Define a recursive sequence $a_n = \sqrt{\frac{1+a_{n-1}}{2}}$, $n > 0$ and $a_0 \in (-1, 1)$. Let $A_n = 4^n(1 - a_n)$ then $\lim_{n\to\infty} A_n =$
(a) $(\cos^{-1}a_0)^2$ (b) $\frac{(\cos^{-1}a_0)^2}{2}$
(c) $\frac{(\cos^{-1}a_0)^2}{3}$ (d) $\frac{(\cos^{-1}a_0)^2}{4}$

9. Given a point (a, b) with $0 < b < a$, the minimum perimeter of a triangle with one vertex at (a, b), one on the x-axis and one on the line $y = x$ is
(a) $\sqrt{a^2+b^2}$ (b) $\sqrt{2(a^2+b^2)}$
(c) $\sqrt{\frac{a^2+b^2}{2}}$ (d) $\sqrt{ab}$

10. 12 points are arranged on a semi-circle as shown. 5 on the diameter and 7 on the arc. If every pair of these points is joined by a straight line segment, then no-three of these line segments intersect at a common point inside the semi-circle. How many points are there inside the semi-circle where two of these line segments intersect?



By : Tapas Kr. Yogi, Mob : 9533632105.

(a) 240 (b) 360
(c) 420 (d) 560

11. The number of integral points (x, y) on the hyperbola $x^2 - y^2 = (2000)^2$ are
(a) 49 (b) 98 (c) 48 (d) 96

SECTION-2
MULTIPLE CORRECT ANSWER TYPE

12. For positive integers n and k, we have

$$\frac{({}^nC_{n-1})^6 + ({}^{n-2}C_k)^6 + ({}^{n+3}C_{n+1})^3}{3\cdot({}^{n-2}C_k)^2\,({}^{n+3}C_2)} = n^2 \text{ then}$$

(a) $n = 6$ (b) $n = 10$
(c) $k = 2$ (d) $k = 4$

13. Consider the equation

$$\sqrt{x+\sqrt{2x-1}} + \sqrt{x-\sqrt{2x-1}} = k$$

(a) If $k = \sqrt{2}$, then $x \in \left[\frac{1}{2}, 1\right]$

(b) If $k = 1$ then $x = \frac{1}{2}$

(c) If $k = \sqrt{2}$, then $x = \frac{3}{2}$

(d) If $k = 2$ then $x = 3/2$

14. If $2\sin^{-1}x - \cos^{-1}x = \frac{\pi}{2}$, then x is equal to

(a) $\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}}$ (c) $\frac{-\sqrt{3}}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

15. Consider the limit, $L = \lim\limits_{x\to 0}(\cos x)^{|\sin x|^{-\alpha}}$ as α ranges from $(0, \infty)$
(a) $L = 1$ if $0 < \alpha < 2$ (b) $L = e^{-1/2}$ if $\alpha = 2$
(c) $L = 0$ if $\alpha > 2$ (d) None of these

SECTION-3
COMPREHENSION TYPE

Passage-1

Let A be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

16. The number of matrices in A is
(a) 2 (b) 6 (c) 9 (d) None of these

17. The number of matrices A in $\mathscr{A}$, for which the system of equations $A\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$ has a unique solution, is

(a) less than 4
(b) at least 4 but less than 7
(c) at least 7 but less than 10
(d) at least 10

Passage-2

The curve $y = f(x)$ passes through the point (0, 1) and the curve $y = g(x) = \int_{-\infty}^{x} f(t)\,dt$ passes through the point $\left(0, \frac{1}{2}\right)$. The tangents drawn to the curves at the point with equal abscissae intersect on the x-axis. Then

18. $y'(0) =$

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) 1

19. $g(0) =$

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) 1

SECTION-4
MATRIX-MATCH TYPE

20. Consider the ellipse $x^2 + 2y^2 = 2$. Let L be the end of the latus rectum in the first quadrant. The tangent at L to the ellipse meets the right side directrix at T. The normal at L meets the major axis at N and the left side directrix at M.

(i) The area of ΔLNT	(p) 1/5
(ii) The circumradius of ΔLNT	(q) $\frac{1}{\sqrt{2}}$
(iii) The ratio $LN : LT$	(r) 3/4
(iv) The ratio $LN : NM$	(s) $\frac{3}{4\sqrt{2}}$

	(i)	(ii)	(iii)	(iv)
(a)	s	r	p	q
(b)	s	p	r	q
(c)	p	s	r	q
(d)	s	r	q	p

SOLUTIONS

1. **(a) :** According to question, $\tan\alpha$ is rational. Let $\gamma = \beta - \alpha$. So, $\alpha = 2\gamma$. Now, $\tan\beta$ is rational if $\tan\gamma$ is rational.

Let $\tan\gamma = r$ then $\tan 2\gamma = \frac{p}{q} = \frac{2r}{1-r^2}$

i.e. $pr^2 + 2qr - p = 0$.
This equation has rational solutions iff
$D = 4(p^2 + q^2)$ is a perfect square.

2. **(a)** **:** The given sum can be rewritten as

$$\sum_{k=0}^{n} {}^nC_k \cdot \left(\tan^2\frac{x}{2}\right)^k + \sum_{k=0}^{n} {}^nC_k \cdot \left(\frac{2\tan^2(x/2)}{1-\tan^2(x/2)}\right)^k$$

$$= \left(1+\tan^2\frac{x}{2}\right)^n + \left(1+\frac{1-\cos x}{\cos x}\right)^n$$

$$= \sec^{2n}\left(\frac{x}{2}\right) + \sec^n(x)$$

3. **(c):** $x^x = \dfrac{1}{\sqrt{2}}$ can be written as $x\log x = \dfrac{1}{2}\log\left(\dfrac{1}{2}\right)$

i.e. $f(x) = f\left(\dfrac{1}{2}\right)$

where $f(x) = x\log x$, $f'(x) = 1 + \log x$

So, $f(x)$ is decreasing in $(0, 1/e)$ and increasing in $(1/e, \infty)$.

So, $f(x) = f\left(\dfrac{1}{2}\right) \Rightarrow x = \dfrac{1}{2}$ or $\dfrac{1}{4}$

Hence two solutions.

4. **(b)** **:** We rewrite the given equation as

$x^5 - x^3 - 4x^2 - 3x - 2 + \lambda(5x^4 + \alpha x^2 - 8x + \alpha) = 0$

A root of this equation is independent of λ iff it is a common root of the equation $x^5 - x^3 - 4x^2 - 3x - 2 = 0$ and $5x^4 + \alpha x^2 - 8x + \alpha = 0$

First equation has roots $x = 2$, ω and ω^2.

So, for $\alpha = -3$, there are two roots independent of λ, $x_1 = \omega$ and $x_2 = \omega^2$.

5. **(d)** **:** Consider the corresponding equation,

$$\left(\sin x - \frac{\sin y}{2}\right)^2 - \frac{3}{4}\cos^2 y = 0$$

i.e. $\sin x = \sin(y + 60°)$	$\sin x = \sin(y - 60°)$
i.e. $x = y + 60°$ or $120° - y$	$x = y - 60°$ or $240° - y$

So, the area of subset of S is bounded by $x = y + 60°$, $x = 120° - y$ and $x = y - 60°$.

So, the required area is $\dfrac{\pi^2}{6}$.

6. **(b)** **:** Let $A(0, 0, 0)$, $B(0, 0, a)$, $C(p, b, c)$, $D(\alpha, \beta, \gamma)$.

Let projection of C on XY plane is $C' \equiv (p, b, 0)$ and projection of D on XY plane is $D' \equiv (\alpha, \beta, 0)$.

$$\text{Area of } \Delta AD'C' = \frac{1}{2}|\overrightarrow{AD'} \times \overrightarrow{AC'}| = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & \beta & 0 \\ p & b & 0 \end{vmatrix}$$

$\Rightarrow \quad |\alpha b - \beta p| = 2\Delta$ ... (1)

So, volume of tetrahedron $ABCD$,

$$V = \frac{1}{6}[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$$

$$V = \frac{1}{6}\begin{vmatrix} 0 & 0 & a \\ p & b & c \\ \alpha & \beta & \gamma \end{vmatrix} = \frac{a}{6} \times 2\Delta \quad \text{(from eqn. (1))}$$

Hence, $V = \dfrac{a\Delta}{3}$.

7. **(b)** **:** Let $A_1(z_1)$, $A_2(z_2)$ and $A_3(z_3)$ and $A(z)$ be any point on the line A_2A_3 such that $z = az_2 + (1 - a)z_3$.

Let P be the foot of altitude from A_1 on line A_2A_3.

So, $AA_1 \geq A_1P$

i.e. min. $|z - z_1|$ = min. $|az_2 + (1 - a)z_3 - z_1| = A_1P = h$

$$\text{So, } \Delta A_1A_2A_3 = \frac{1}{2}h \cdot |z_2 - z_3| = \frac{1}{2}(A_1A_2)(A_1A_3)(\sin A_1)$$

$$= \frac{1}{2}|z_1 - z_2||z_1 - z_3| \cdot \frac{|z_2 - z_3|}{2r}$$

$$\text{So, } h = \frac{|z_1 - z_2||z_1 - z_3|}{2r}$$

8. **(b)** **:** Let $a_0 = \cos\theta$, $\theta \in (0, \pi)$ then $a_1 = \cos(\theta/2)$

Similarly, $a_2 = \cos(\theta/4)$, ..., $a_n = \cos(\theta/2^n)$

So, $A_n = 4^n[1 - \cos(\theta/2^n)]$

$$= \frac{\theta^2}{1+\cos(\theta/2^n)} \cdot \left[\frac{\sin(\theta/2^n)}{\theta/2^n}\right]^2$$

$$\text{So, as } n \to \infty,\ A_\infty = \frac{\theta^2}{2}(1) = \frac{\theta^2}{2} = \frac{1}{2}(\cos^{-1} a_0)^2$$

9. **(b)** **:** Let $A(a, b)$, $B(x, 0)$ and let $E(b, a)$ be the reflection of $A(a, b)$ in the line $y = x$, $D(a, -b)$ be the reflection of A in x-axis. Let C be the point on $y = x$ line then $AB = DB$ and $AC = CE$.

So, perimeter of $\Delta ABC = AB + BC + CA$

$= DB + BC + CE \geq DE$

$$\geq \sqrt{(a-b)^2 + (a+b)^2} = \sqrt{2(a^2+b^2)}$$

10. **(c):** 5 points on diameter line and 7 points on arc.

Two line segments can intersect at an interior point in following ways.

(i) Line segments have all 4 end points on the arc

$= {}^7C_4 = 35$

MPP CLASS XI ANSWER **KEY**

1	(a)	2	(b)	3	(c)	4	(a)	5	(d)
6.	(b)	7.	(a, b, c)	8.	(a, c, d)	9 .	(a, b, d)	10.	(a, d)
11.	(a, b, c, d)			12	(a, b)	13	(a, b, c)	14	(a)
15	(b)	16.	(b)	17.	(1)	18.	(2)	19.	(5)
20.	(5)								

(ii) Each line segment has one end point on diameter and one end point on arc = ${}^7C_2 \times {}^5C_2 = 210$.

(iii) One line segment has both end points on the arc and the other has 1 end point on diameter and one on the arc = ${}^7C_3 \times {}^5C_1 = 175$

Total = 35 + 210 + 175 = 420.

11. (b) : $(x + y)(x - y) = 2^8 \cdot 5^6$

So, $(x + y)$ and $(x - y)$ both are even.

Let us first assign a factor of 2 to both $(x + y)$ and $(x - y)$. We have $2^6 \times 5^6$ left.

Since there are $(6 + 1)\cdot(6 + 1) = 49$ factors of $2^6 \times 5^6$ and since both x and y can be negative. So, total integral points = 2 × 49 = 98

12. (a, c): Use A.M. ≥ G.M. on $({}^nC_{n-1})^6 \cdot ({}^{n-2}C_k)^6$ and $({}^{n+3}C_2)^3$ with equality occuring when the numbers are equal.

13. (a, d) : Let $\sqrt{2x-1} = t$ then $k = \left|\frac{t+1}{\sqrt{2}}\right| + \left|\frac{t-1}{\sqrt{2}}\right|$

14. (d) : $2\sin^{-1} x - \cos^{-1} x = \pi/2$ ...(i)

Also, $\sin^{-1} x + \cos^{-1} x = \pi/2$ ...(ii)

Adding (i) and (ii), $3\sin^{-1} x = \pi$

$\Rightarrow \sin^{-1} x = \pi/3 \Rightarrow x = \sin(\pi/3) \Rightarrow x = \sqrt{3}/2$

15. (a, b, c) : The given limit is

$$e^{\lim\limits_{x\to 0}\left(\frac{1}{|\sin x|^{\alpha}}\cdot \log(\cos x)\right)}$$

$$= e^{\lim\limits_{x\to 0}\left(\frac{x^2}{|\sin x|^{\alpha}}\cdot\frac{\cos x - 1}{x^2}\cdot\frac{\log(\cos x)}{\cos x - 1}\right)}$$

= 1 for $0 < \alpha < 2$

= $e^{-1/2}$ for $\alpha = 2$

= 0 for $\alpha > 2$

16. (d): $\begin{bmatrix} 1 & h & g \\ h & 1 & f \\ g & f & 1 \end{bmatrix}$ where only one of f, g, h is 1, (3 ways)

$\begin{bmatrix} 1 & h & g \\ h & 0 & f \\ g & f & 0 \end{bmatrix}$ where only one of f, g, h is 0. (3 ways)

$\begin{bmatrix} 0 & h & g \\ h & 1 & f \\ g & f & 0 \end{bmatrix}$ (3 ways), $\begin{bmatrix} 0 & h & g \\ h & 0 & f \\ g & f & 1 \end{bmatrix}$ (3 ways)

∴ Required number of matrices = 12 ways.

17. (b) : $D = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = abc + 2fgh - af^2 - bg^2 - ch^2$

$a = b = 0, c = 1 \Rightarrow D \neq 0$ if $g = 0$ or $f = 0$, 2 matrices

$a = c = 0, b = 1 \Rightarrow D \neq 0$ if $f = 0$ or $h = 0$, 2 matrices

$b = c = 0, a = 1 \Rightarrow D \neq 0$ if $g = 0$ or $h = 0$, 2 matrices

$a = b = c = 1 \Rightarrow D = 0$ if f or g or h is 1.

The number of matrices is 6.

18. (c) **19. (b)**

20. (d) : (i) → (s), (ii) → (r), (iii) → (q), (iv) → (p)

MATH archives

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of IIT-JEE Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for IIT-JEE. In every issue of MT, challenging problems are offered with detailed solution. The readers' comments and suggestions regarding the problems and solutions offered are always welcome.

1. If $|z-1|+|z+3| \le 8$, then the range of values of $|z-4|$ is (z is a complex number in the argand plane)

(a) $(0, 8)$ (b) $[0, 8]$ (c) $[1, 9]$ (d) $[5, 9]$

2. The value of $\sum_{k=1}^{n}(n-k)\cos\frac{2k\pi}{n}\,(n \ge 3)$ is

(a) $-\frac{n}{2}$ (b) 0 (c) $\frac{n}{2}$ (d) $-n$

3. Let $g(x)=\frac{f(x)}{x+1}$ where $f(x)$ is differentiable on $[0, 5]$ such that $f(0) = 4$, $f(5) = -1$. There exists $c \in (0, 5)$ such that $g'(c)$ is

(a) $-\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $-\frac{5}{6}$ (d) -1

4. If $4x^4 + 9y^4 = 64$, then the maximum value of $x^2 + y^2$ is (where x and y are real)

(a) $\frac{4}{\sqrt{3}}$ (b) $\frac{4}{3}\sqrt{13}$ (c) $\frac{32}{3}$ (d) $\frac{32}{13}$

5. A chord of the circle $x^2 + y^2 - 4x - 6y = 0$ passing through origin subtends an angle $\tan^{-1}(7/4)$ at the point where the circle meets positive y-axis, then equation of the chord is

(a) $2x + 3y = 0$ (b) $x + 2y = 0$
(c) $x - 2y = 0$ (d) $2x - 3y = 0$

6. The number of points of extremum of the function $f(x) = (x-2)^{2/3}(2x+1)$ is

(a) 1 (b) 0 (c) 2 (d) 3

7. Let the vector $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ be vertical. The line of greatest slope on a plane with normal $\vec{b}=2\hat{i}-\hat{j}+\hat{k}$ is along the vector _______.

(a) $\hat{i}-4\hat{j}+2\hat{k}$ (b) $\hat{i}+4\hat{j}-3\hat{k}$
(c) $4\hat{i}+\hat{j}-\hat{k}$ (d) none of these

8. The probability that the triangle formed by choosing any three vertices from the vertices of a cube is equilateral, is

(a) 3/7 (b) 6/7 (c) 4/7 (d) 1/7

9. Let $A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22}\end{bmatrix}$, $X=\begin{bmatrix} x_1 \\ x_2\end{bmatrix}$, $Y=\begin{bmatrix} y_1 \\ y_2\end{bmatrix}$ (X', Y' denote the transpose of X and Y).

Statement-1 : If A is symmetric, then $X'AY = Y'AX$ for each pair of X and Y.

Statement-2 : If $X'AY = Y'AX$ for each pair of X and Y, then A is symmetric.

Which of the following options holds?

(a) Only Statement-1 is true.
(b) Only Statement-2 is true.
(c) Both Statement-1 and Statement-2 are true.
(d) Both Statement-1 and Statement-2 are false.

10. Consider the following relations:

$R = \{(x, y)/x, y$ are real numbers and $x = wy$ for some rational number $w\}$

$S=\left\{\left(\frac{m}{n},\frac{p}{q}\right)\right\}$ m, n, p, q are integers such that $nq \ne 0$ and $qm = pn\}$

Statement-1 : S is an equivalence relation but R is not an equivalence relation.

Statement-2 : R and S both are symmetric.

(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1.

By : **Prof. Shyam Bhushan,** Director, Narayna IIT Academy, Jamshedpur. Mob. : 09334870021

(b) Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-1.

(c) Statement-1 is true, Statement-2 is false.

(d) Statement-1 is false, Statement-2 is true.

SOLUTIONS

1. (c) : z lies on or inside an ellipse with foci (–3, 0) and (1, 0) and (4, 0) is a point outside the ellipse. Its minimum and maximum distances from any point on the ellipse are 1 and 9.

2. (a) : Let $S=(n-1)\cos\frac{2\pi}{n}+(n-2)\cos\frac{4\pi}{n}+(n-3)\cos\frac{6\pi}{n}+......+\cos\frac{2(n-1)\pi}{n}$ ...(i)

$\Rightarrow\ S=1\cos\frac{2\pi}{n}+2\cos\frac{4\pi}{n}+.....+(n-1)\cos\frac{2(n-1)\pi}{n}$ ...(ii)

Adding (i) and (ii), we get

$$2S=n\left(\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}+.....+\cos\frac{2(n-1)\pi}{n}\right)$$

$$\Rightarrow\ 2S=n\frac{\sin(n-1)\frac{\pi}{n}}{\sin\frac{\pi}{n}}\cos\left(\frac{\frac{2\pi}{n}+\frac{2(n-1)\pi}{n}}{2}\right)=-n$$

3. (c) : $g'(c)=\dfrac{\frac{f(5)}{6}-\frac{f(0)}{1}}{5}$ (from L.M.V.T)

$$=\frac{-\frac{1}{6}-4}{5}=\frac{-25}{6\times5}=-\frac{5}{6}$$

4. (b) : Let $x^2 = 4\sin\theta,\ y^2=\frac{8}{3}\cos\theta$

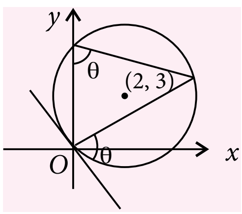
$\Rightarrow\ x^2+y^2=4\sin\theta+\frac{8}{3}\cos\theta$

$\therefore$ Maximum value $=\sqrt{16+\frac{64}{9}}=\frac{4\sqrt{13}}{3}$

5. (c) : Equation of tangent at origin is

$-2(x + 0) - 3(y + 0) = 0$

$\Rightarrow\ 2x + 3y = 0$



$\tan\theta=\frac{7}{4}\ \Rightarrow\left|\dfrac{m+\frac{2}{3}}{1-\frac{2m}{3}}\right|=\frac{7}{4}$

$\Rightarrow\ \dfrac{3m+2}{3-2m}=\pm\frac{7}{4}\ \Rightarrow\ 12m+8=\pm(21-14m)$

$\Rightarrow\ 26m=13\ \Rightarrow\ m=\frac{1}{2}$ or $2m=29\ \Rightarrow\ m=\frac{29}{2}$

$\therefore\ y=\frac{1}{2}x\Rightarrow x-2y=0$ =

or $y=\frac{29}{2}x\ \Rightarrow\ 29x-2y=0$

6. (c) : $f'(x)=(x-2)^{2/3}2+(2x+1)\frac{2}{3}(x-2)^{-1/3}$

$$=\frac{6(x-2)+2(2x+1)}{3(x-2)^{1/3}}=\frac{10x-10}{3(x-2)^{1/3}}$$

x = 1 is a point of maximum and x = 2 is a point of minimum.

$\therefore$ Number of extremum points is 2.

7. (d)

8. (d) : $n(S)={}^8C_3=56$

Choosing a vertex 3 triangles can be formed from the diagonals of the faces not passing through the vertex. But each triangle is repeated 3 times.

$\therefore\ n(E)=\frac{8\times3}{3}=8$ $\therefore$

$\therefore\ P(E)=\frac{8}{56}=\frac{1}{7}$

9. (c) : A is symmetric $\Rightarrow A'=A$

Now, $X'AY=((X'AY)')$ ($\because$ $X'AY$ is 1×1 matrix)

$=(Y'A'X)=(Y'AX)$

Let $E_1=\begin{bmatrix}1\\0\end{bmatrix},\ E_2=\begin{bmatrix}0\\1\end{bmatrix}$

If $X=E_1$ and $Y=E_2$ then

$E_1'AE_2=E_2'AE_1\ \Rightarrow\ a_{12}=a_{21}$

$\therefore$ A is symmetric.

10. (c) : For $(x, y)\in R\ \Rightarrow x=wy$

but $(y, x)\in R\ \Rightarrow\ y=xw\ \Rightarrow\ x=y$

$\therefore$ R is not symmetric.

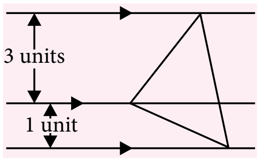
MPP CLASS XII ANSWER KEY

1.	(c)	2.	(b)	3.	(a)	4.	(a)	5.	(c)
6.	(d)	7.	(b, d)	8.	(a, b)	9.	(a, b, c, d)		
10.	(b, d)	11.	(a, b, c)	12.	(a, d)	13.	(a, b, c, d)		
14.	(a)	15.	(b)	16.	(a)	17.	(3)	18.	(5)
19.	(5)	20.	(1)						

OLYMPIAD CORNER

1. Let $\frac{3}{4} < a < 1$. Prove that the equation $x^3(x + 1) = (x + a)(2x + a)$ has four distinct real solutions and find these solutions in explicit form.

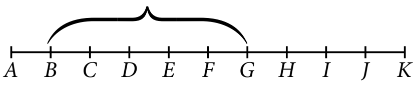
2. The vertices of an equilateral triangle lie on three parallel lines which are 3 units and 1 unit apart as shown. Find the area of this triangle.



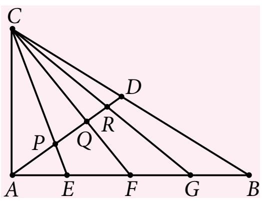
3. Find all real numbers x such that

$$x = \left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2}$$

4. A railway line is divided into 10 sections by the stations A, B, C, D, E, F, G, H, I, J and K. The distance between A and K is 56 km. A trip along two successive sections never exceeds 12 km. A trip along three successive sections is at least 17 km. What is the distance between B and G?



5. Given is a triangle ABC, $\angle A = 90°$. D is the midpoint of BC, F is the midpoint of AB, E the midpoint of AF and G the midpoint of FB. AD intersects CE, CF and CG respectively in P, Q and R. Determine the ratio $\frac{PQ}{QR}$.



SOLUTIONS

1. Look at the given equation as a quadratic equation in a:
$a^2 + 3xa + 2x^2 - x^3 - x^4 = 0$
The discriminant of this equation is
$9x^2 - 8x^2 + 4x^3 + 4x^4 = (x + 2x^2)^2$

Thus $a = \frac{-3x \pm (x + 2x^2)}{2}$

The first choice $a = -x + x^2$ yields the quadratic equation $x^2 - x - a = 0$, whose solutions are

$x = \frac{(1 \pm \sqrt{1+4a})}{2}$

The second choice $a = -2x - x^2$ yields the quadratic equation $x^2 + 2x + a = 0$,
whose solutions are $-1 \pm \sqrt{1-a}$.
The inequalities

$$-1 - \sqrt{1-a} < -1 + \sqrt{1-a} < \frac{1-\sqrt{1+4a}}{2} < \frac{1+\sqrt{1+4a}}{2}$$

show that the four solutions are distinct.

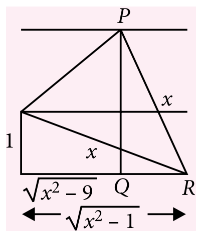
Indeed $-1 + \sqrt{1-a} < \frac{1-\sqrt{1+4a}}{2}$

reduces to $2\sqrt{1-a} < 3 - \sqrt{1+4a}$

which is equivalent to $6\sqrt{1+4a} < 6 + 8a$

or $3a < 4a^2$, which is evident.

2. Using Pythagoras' Theorem on the triangle marked PQR we have



$$4^2 + \left(\sqrt{x^2-1} - \sqrt{x^2-9}\right)^2 = x^2$$

i.e., $16 + x^2 - 1 + x^2 - 9$

$$-2\sqrt{(x^2-1)(x^2-9)} = x^2$$

This rearranges to give

$$x^2+6=2\sqrt{(x^2-1)(x^2-9)},$$
which, on squaring both sides, becomes

$x^4 + 12x^2 + 36 = 4(x^4 - 10x^2 + 9) = 4x^4 - 40x^2 + 36$

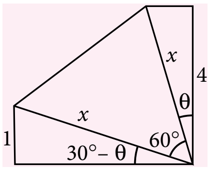
This simplifies to $3x^4 - 52x^2 = 0$, or $x^2 = 52/3$. The area of an equilateral triangle of side length x is half base times height, *i.e.*, $1/2 \times x \times \sqrt{3}x/2$ or $x^2 \times \sqrt{3}/4$. The area of this triangle is thus $52\sqrt{3}/12 = 13\sqrt{3}/3$

Alternative method:

Let the side lengths of the equilateral triangle be x and the angle θ be as marked in the diagram.

First we note that $\cos\theta = 4/x$ while

$$\sin\theta = \sqrt{1-\frac{16}{x^2}}.$$



Further,

$$\sin(30^\circ - \theta) = \frac{\cos\theta}{2} - \frac{\sqrt{3}\sin\theta}{2}$$
$$= \frac{1}{x} = \frac{\cos\theta}{4}.$$

So $\cos\theta = 2\sqrt{3}\sin\theta$ or $\cos^2\theta = 12\sin^2\theta = 12 - 12\cos^2\theta$ giving $13\cos^2\theta = 12$ or $\cos^2\theta = 12/13$. So

$$x^2 = \frac{16}{\cos^2\theta} = \frac{(16)(13)}{12} = \frac{4(13)}{3}.$$

The required area is

$$\frac{1}{2}x^2\sin 60^\circ = \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4}\cdot 4\cdot\frac{13}{3} = \frac{13\sqrt{3}}{3},$$

3. Since $\left(x-\frac{1}{x}\right)^{1/2} \geq 0$ and $\left(1-\frac{1}{x}\right)^{1/2} \geq 0$, then

$$0 \leq \left(x-\frac{1}{x}\right)^{1/2} + \left(1-\frac{1}{x}\right)^{1/2} = x$$

Note that $x \neq 0$. Else, $\frac{1}{x}$ would not be defined, so $x > 0$.

Squaring both sides gives,

$$x^2 = \left(x-\frac{1}{x}\right) + \left(1-\frac{1}{x}\right) + 2\sqrt{\left(x-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)}$$

$$\Rightarrow x^2 = x + 1 - \frac{2}{x} + 2\sqrt{x - 1 - \frac{1}{x} + \frac{1}{x^2}}$$

Multiplying both sides by x and rearranging, we get

$$x^3 - x^2 - x + 2 = 2\sqrt{x^3 - x^2 - x + 1}$$

$$\Rightarrow (x^3 - x^2 - x + 1) - 2\sqrt{x^3 - x^2 - x + 1} + 1 = 0$$

$$\Rightarrow (\sqrt{x^3 - x^2 - x + 1} - 1)^2 = 0 \Rightarrow \sqrt{x^3 - x^2 - x + 1} = 1$$

$$\Rightarrow x^3 - x^2 - x + 1 = 1 \Rightarrow x(x^2 - x - 1) = 0$$

$$\Rightarrow x^2 - x - 1 = 0 \qquad \text{(Since } x \neq 0\text{)}$$

Thus $x = \frac{1 \pm \sqrt{5}}{2}$. We must check to see if these are indeed solutions.

Let $\alpha = \frac{1+\sqrt{5}}{2}, \beta = \frac{1-\sqrt{5}}{2}$. Note that $\alpha + \beta = 1$, $\alpha\beta = -1$ and $\alpha > 0 > \beta$.

Since $\beta < 0$, β is not a solution.

Now, if $x = \alpha$, then

$$\left(\alpha - \frac{1}{\alpha}\right)^{1/2} + \left(1 - \frac{1}{\alpha}\right)^{1/2}$$

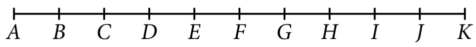
$= (\alpha + \beta)^{1/2} + (1 + \beta)^{1/2}$ (Since $\alpha\beta = -1$)

$= 1^{1/2} + (\beta^2)^{1/2}$ (Since $\alpha + \beta = 1$ and $\beta^2 = \beta + 1$)

$= 1 - \beta$ (Since $\beta < 0$)

$= \alpha$ (Since $\alpha + \beta = 1$)

So $x = \alpha$ is the unique solution to the equation.

4. 

Now $\overline{AK} = 56$ and $\overline{AK} = \overline{AD} + \overline{DG} + \overline{GJ} + \overline{JK}$. We know that $\overline{AD}, \overline{DG}, \overline{GJ} \geq 17$. Thus $\overline{JK} \leq 5$ to satisfy $\overline{AK} = 56$. We know $\overline{HK} \geq 17$, and since $\overline{JK} \leq 5$, $\overline{HJ} \geq 12$. But, we also know $\overline{HJ} \leq 12$. Thus $\overline{HJ} = 12$. Since $\overline{HK} \geq 17$ and $\overline{HJ} = 12$, $\overline{JK} \geq 5$. The only possibility is that $\overline{JK} = 5$.

Symmetrically we find that $\overline{AB} = 5$ and $\overline{BD} = 12$.

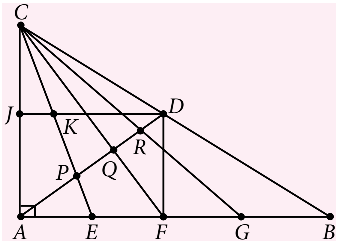
Now, $\overline{DH} = \overline{AK} - \overline{AB} - \overline{BD} - \overline{HJ} - \overline{JK}$

$= 56 - 5 - 12 - 5 - 12 = 22$

Now $\overline{GJ} \geq 17$ but $\overline{HJ} = 12$. Hence $\overline{GH} \geq 5$. Since $\overline{DG} \geq 17$ and $\overline{DH} = \overline{DG} + \overline{GH} = 22$, we obtain $\overline{DG} = 17$ and $\overline{GH} = 5$.

Now, $\overline{BG} = \overline{BD} + \overline{DG} = 12 + 17 = 29$

5. We know that two medians in a triangle divide each other in 2 : 1 ratio, or in other words the point of intersection is $\frac{2}{3}$ the way from the vertex.



Since CF and AD are both medians in ΔABC, then $\frac{\overline{AQ}}{\overline{QD}}=\frac{2}{1}$, where Q is the point of intersection.

Also, since D is the midpoint of the hypotenuse in the right triangle ABC, then it is the centre of the circumscribed circle with radius DA = DC = DB.

Drop a perpendicular from D onto sides AB and CA. The feet of the perpendiculars will be F and J, respectively, where J is the midpoint of AC, since DF and DJ are altitudes in isosceles triangles ΔADB and ΔADC, respectively. Now consider ΔCFB. The segments CG and FD are medians and therefore intersect at H say in the ratio 2 : 1 so, $\frac{\overline{HD}}{\overline{FD}}=\frac{1}{3}$. From here it can be seen that ΔARC and ΔDRH are similar, since their angles are the same. Also, since we know that $\overline{FD}=\overline{JA}$, and $2\overline{JA}=\overline{AC}$ then $\overline{HD}=\frac{1}{6}\overline{CA}$ and ΔARC is 6 times bigger than ΔDRH. Now we can see that $\frac{\overline{AR}}{\overline{RD}}=\frac{6}{1}$ and since $\overline{AR}+\overline{RD}=\overline{AD}$, then $\frac{\overline{RD}}{\overline{AD}}=\frac{1}{7}$.

Similarly $\Delta APE \sim \Delta KPD$, where medians DJ and CE meet at K. We know that $\overline{AE}=\frac{1}{4}\overline{AB}$, so then $\overline{JK}=\frac{1}{4}\overline{JD}$, since JD is parallel to AB. It now follows that $\frac{\overline{AE}}{\overline{KD}}=\frac{2}{3}$, and from the similarity of the triangles $\frac{\overline{AP}}{\overline{PD}}=\frac{2}{3}$. Also, since $\overline{AP}+\overline{PD}=\overline{AD}$, then $\frac{\overline{AP}}{\overline{AD}}=\frac{2}{5}$. Combining these results we have

$$\overline{AP}=\frac{2}{5}\overline{AD},\ \overline{AQ}=\frac{2}{3}\overline{AD},\ \overline{QD}=\frac{1}{3}\overline{AD}$$

and $\overline{RD}=\frac{1}{7}\overline{AD}$.

Thus $\overline{PQ}=\overline{AQ}-\overline{AP}=\frac{2}{3}\overline{AD}-\frac{2}{5}\overline{AD}=\frac{4}{15}\overline{AD}$

and $\overline{QR}=\overline{QD}-\overline{RD}=\frac{1}{3}\overline{AD}-\frac{1}{7}\overline{AD}=\frac{4}{21}\overline{AD}$

From these $\frac{\overline{PQ}}{\overline{QR}}=\frac{7}{5}$.

QUANTITATIVE APTITUDE

Useful for Bank PO, Specialist Officers & Clerical Cadre, BCA, MAT, CSAT, CDS and other such examinations

1. The maximum number of students among them 1001 pens and 910 pencils can be distributed in such a way that each student gets the same number of pens and same number of pencils is
   (a) 91 (b) 910 (c) 1001 (d) 1911

2. Each boy contributed rupees equal to the number of girls and each girl contributed rupees equal to the number of boys in a class of 60 students. If the total contribution thus collected is ₹1600, how many boys are there in the class?
   (a) 25 (b) 30
   (c) 50 (d) Data inadequate

3. The average temperature of the town in the first four days of a month was 58 degrees. The average for the 2nd, 3rd, 4th and 5th days was 60 degrees. If the temperature of the 1st and 5th days were in the ratio 7 : 8, then what is the temperature on the 5th day?
   (a) 64 degrees (b) 62 degress
   (c) 56 degrees (d) None of these

4. The sum of four numbers is 64. If you add 3 to the first number, 3 is subtracted from the second number, the third is multiplied by 3 and the fourth is divided by 3, then all the results are equal. What is the difference between the largest and the smallest of the original numbers?
   (a) 21 (b) 27
   (c) 32 (d) None of these

5. A spider climbed $62\frac{1}{2}\%$ of the height of the pole in one hour and in the next hour it covered $12\frac{1}{2}\%$ of the remaining height. If the height of the pole is 192 m, then distance climbed in second hour is
   (a) 3 m (b) 5 m (c) 7 m (d) 9 m

6. Samant bought a microwave oven and paid 10% less than the original price. He sold it with 30% profit on the price he had paid. What percentage of profit did Samant earn on the original price?
   (a) 17% (b) 20% (c) 27% (d) 32%

7. The electricity bill of a certain establishment is partly fixed and partly varies as the number of units of electricity consumed. When in a certain month 540 units are consumed, the bill is ₹1800. In another month 620 units are consumed and the bill is ₹2040. In yet another month 500 units are consumed. The bill for that month would be
   (a) ₹1560 (b) ₹1840 (c) ₹1680 (d) ₹1950

8. Which of the following statements is/are necessary to answer the given question.
   Three friends, *P*, *Q* and *R* started a partnership business investing money in the ratio of 5 : 4 : 2 respectively for a period of 3 years. What is the amount received by *P* as his share in the total profit?
   Statement (I) : Total amount invested in the business in ₹22,000.
   Statement (II) : Profit earned at the end of 3 years is $\frac{3}{8}$ of the total investment.
   Statement (III) : The average amount of profit earned per year is ₹2750.
   (a) Statement (I) or Statement (II) or Statement (III)
   (b) Either Statement (III) only, or Statement (I) and Statement (II) together
   (c) Any two of the three
   (d) All Statement (I), Statement (II) and Statement (III)

9. Ronald and Elan are working on an assignment. Ronald takes 6 hours to type 32 pages on a computer, while Elan takes 5 hours to type 40 pages. How much time will they take, working together on two different computers to type an assignment of 110 pages?

(a) 7 hours 30 minutes
(b) 8 hours
(c) 8 hours 15 minutes
(d) 8 hours 25 minutes

10. A person borrowed ₹500 @ 3% per annum S.I. and ₹600 @ $4\frac{1}{2}$% per annum on the agreement that the whole sum will be returned only when the total interest becomes ₹126. The number of years, after which the borrowed sum is to be returned is

(a) 2 (b) 3
(c) 4 (d) 5

11. A park square in shape has a 3 metre wide road inside it running along its sides. The area occupied by the road is 1764 square metres. What is the perimeter along the outer edge of the road ?

(a) 576 metres (b) 600 metres
(c) 640 metres (d) Data inadequate

12. Two metallic right circular cones having their heights 4.1 cm and 4.3 cm and the radii of their bases 2.1 cm each, have been melted together and recast into a sphere. Find the diameter of the sphere.

(a) 7.1 cm (b) 4.2 cm
(c) 3.1 cm (d) 6.4 cm

13. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

(a) $\frac{2}{5}$ (b) $\frac{3}{4}$
(c) $\frac{3}{5}$ (d) $\frac{3}{10}$

Direction (14-16) : Study the table and answer the given questions.

Expenditures of a company (in Lakh rupees) per annum over the given years.

Years \ Items of expenditure	Salary	Fuel and Transport	Bonus	Interest on Loans	Taxes
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

14. The ratio between the total expenditure on Taxes for all the years and the total expenditure on Fuel and Transport for all the years respectively is approximately.

(a) 4 : 7 (b) 10 : 13
(c) 15 : 18 (d) 5 : 8

15. What is the average amount of interest per year which the Company had to pay during this period?

(a) ₹32.43 lakhs (b) ₹33.72 lakhs
(c) ₹34.18 lakhs (d) ₹36.66 lakhs

16. Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002 ?

(a) 62% (b) 66% (c) 69% (d) 71%

17. A booster pump can be used for filling as well as for emptying a tank. The capacity of the tank is 2400 m^3. The emptying capacity of the tank is 10 m^3 per minute higher than its filling capacity and the pump needs 8 minutes lesser to empty the tank than it needs to fill it. What is the filling capacity of the pump ?

(a) 50 m^3/min. (b) 60 m^3/min.
(c) 72 m^3/min. (d) None of these

18. The number of students in each section of a school is 24. After admitting new students, three new sections were started. Now, the total number of sections is 16 and there are 21 students in each section. The number of new students admitted is

(a) 14 (b) 24 (c) 48 (d) 114

19. 4 mat-weavers can weave 4 mats in 4 days. At the same rate, how many mats would be woven by 8 mat-weavers in 8 days ?

(a) 4 (b) 8 (c) 12 (d) 16

20. In dividing a number by 585, a student employed the method of short division. He divided the number successively by 5, 9 and 13 (factors of 585) and got the remainders 4, 8, 12 respectively. If he had divided the number by 585, the remainder would have been

(a) 24 (b) 144 (c) 292 (d) 584

SOLUTIONS

1. **(a) :** Required number of students = H.C.F. of 1001, and 910 = 91.

2. **(d):** Let number of boys = x
Then, number of girls = $(60 - x)$
$\therefore$ $x(60 - x) + (60 - x)x = 1600$
$\Rightarrow 2x^2 - 120x + 1600 = 0 \Rightarrow x^2 - 60x + 800 = 0$
$\Rightarrow (x - 40)(x - 20) = 0 \Rightarrow x = 40$ or $x = 20$
Hence, data is inadequate.

3. **(a) :** Sum of temperature on 1st, 2nd, 3rd and 4th days $= (58 \times 4) = 232$ degrees ...(i)
Sum of temperature on 2nd, 3rd, 4th and 5th days $= (60 \times 4) = 240$ degrees ...(ii)
Subtracting (i) from (ii), we get
Temp. on 5th day – Temp. on 1st day = 8 degrees
Let the temperature on 1st and 5th days be $7x$ and $8x$ degrees respectively.
Then, $8x - 7x = 8 \Rightarrow x = 8$
$\therefore$ Temperature on the 5th day $= 8x = 64$ degrees.

4. **(c) :** Let the four numbers be A, B, C and D. Let
$A + 3 = B - 3 = 3C = \frac{D}{3} = x$
Then, $A = x - 3, B = x + 3, C = \frac{x}{3}$ and $D = 3x$
$A + B + C + D = 64 \Rightarrow (x-3) + (x+3) + \frac{x}{3} + 3x = 64$
$\Rightarrow 16x = 192 \Rightarrow x = 12$
Thus, the numbers are 9, 15, 4 and 36
$\therefore$ Required difference $= (36 - 4) = 32$.

5. **(d):** Height climbed in second hour $= 12\frac{1}{2}\%$ of $\left(100 - 62\frac{1}{2}\right)\%$ of 192 m
$= \left(\frac{25}{2} \times \frac{1}{100} \times \frac{75}{2} \times \frac{1}{100} \times 192\right)$ m = 9 m

6. **(a) :** Let original price = ₹ 100
Then, C.P. = ₹ 90
S.P. = 130% of ₹ 90 = ₹ $\left(\frac{130}{100} \times 90\right)$ = ₹ 117
$\therefore$ Required percentage $= (117 - 100)\% = 17\%$

7. **(c) :** Let the fixed amount be ₹ x and the cost of each unit be ₹ y. Then,
$540y + x = 1800$ ...(i)
$620y + x = 2040$ ...(ii)
On subtracting (i) from (ii), we get
$80y = 240 \Rightarrow y = 3$
Putting $y = 3$ in (i), we get
$540 \times 3 + x = 1800 \Rightarrow x = 180$
$\therefore$ Fixed amount = ₹ 180, Cost per unit = ₹ 3
Total cost for consuming 500 units
= ₹ (180 + 500 × 3) = ₹ 1680.

8. **(b):** Statement (I) and Statement (II) give, profit after 3 years = ₹ $\left(\frac{3}{8} \times 22000\right)$ = ₹ 8250
From Statement (III) also, profit after 3 years
= ₹ (2750 × 3) = ₹ 8250
$\therefore$ P's share = ₹ $\left(8250 \times \frac{5}{11}\right)$ = ₹ 3750
Thus, (either Statement (III) is redundant) or (Statement (I) and Statement (II) are redundant).
$\therefore$ Correct answer is (b).

9. **(c) :** Number of pages typed by Ronald in 1 hour
$= \frac{32}{6} = \frac{16}{3}$
Number of pages typed by Elan in 1 hour $= \frac{40}{5} = 8$
Number of pages typed by both in 1 hour
$= \left(\frac{16}{3} + 8\right) = \frac{40}{3}$
$\therefore$ Time taken by both to type 110 pages
$= \left(110 \times \frac{3}{40}\right)$ hours $= 8\frac{1}{4}$ hours = 8 hours 15 minutes.

10. **(b):** Let the time be x years
Then, $\left(\frac{500 \times 3 \times x}{100}\right) + \left(\frac{600 \times 9 \times x}{100 \times 2}\right) = 126$
$\Rightarrow 15x + 27x = 126 \Rightarrow x = 3$
$\therefore$ Required time = 3 years.

11. **(b):** Let the length of the outer edge be x m.
Then length of the inner edge $= (x - 6)$ m
$\therefore$ $x^2 - (x-6)^2 = 1764 \Rightarrow x^2 - (x^2 - 12x + 36) = 1764$
$\Rightarrow 12x = 1800 \Rightarrow x = 150$
$\therefore$ Required perimeter $= (4x)$ m $= (4 \times 150)$ m $= 600$ m.

12. (b): Volume of sphere = Volume of 2 cones

$$= \left[\frac{1}{3}\pi\times(2.1)^2\times 4.1+\frac{1}{3}\pi\times(2.1)^2\times 4.3\right]\text{cm}^3$$

$$= \frac{1}{3}\pi\times(2.1)^2(8.4)\text{cm}^3$$

Let the radius of the sphere be R.

$$\therefore\ \frac{4}{3}\pi R^3=\frac{1}{3}\pi(2.1)^3\times 4 \quad\Rightarrow\quad R=2.1\text{ cm}$$

Hence, diameter of the sphere = 4.2 cm.

13. (a): $P(\text{E})=\frac{30}{100}=\frac{3}{10},\ P(\text{H})=\frac{20}{100}=\frac{1}{5}$

and $P(\text{E}\cap\text{H})=\frac{10}{100}=\frac{1}{10}$

P (E or H) = P (E ∪ H)

$= P(\text{E}) + P(\text{H}) - P(\text{E}\cap\text{H})$

$$= \left(\frac{3}{10}+\frac{1}{5}-\frac{1}{10}\right)=\frac{4}{10}=\frac{2}{5}.$$

14. (b): Required ratio $=\dfrac{(83+108+74+88+98)}{(98+112+101+133+142)}$

$$=\frac{451}{586}\approx\frac{1}{1.3}=10:13$$

15. (d): Average amount of interest paid by the company during the given period

$$= ₹\left(\frac{23.4+32.5+41.6+36.4+49.4}{5}\right)\text{ lakhs}$$

$$= ₹\left(\frac{183.3}{5}\right)\text{ lakhs} = ₹\ 36.66 \text{ lakhs}$$

16. (c): Required percentage

$$=\left[\frac{288+98+3.00+23.4+83}{420+142+3.96+49.4+98}\times 100\right]\%$$

$$=\left(\frac{495.4}{713.36}\times 100\right)\%\approx 69.45\%\approx 69\%\text{ (approx.)}$$

17. (a): Let the filling capacity of the pump be x m^3/min.
Then, emptying capacity of the pump

$= (x + 10)$ m^3/min

According to question, $\dfrac{2400}{x}-\dfrac{2400}{(x+10)}=8$

$\Rightarrow\ x^2 + 10x - 3000 = 0$

$(x - 50)(x + 60) = 0 \quad\Rightarrow\quad x = 50$

[neglecting the – ve value of x]

So, filling capacity of the pump = 50 m^3/min.

18. (b): Original number of sections = (16 – 3) = 13

Original number of students = (24 × 13) = 312
Present number of students = (21 × 16) = 336
∴ Number of new students admitted = (336 – 312)
= 24

19. (d): Let the required number of mats be x.

More weavers, More mats (Direct Proportion)
More days, More mats (Direct Proportion)

$$\left.\begin{array}{ll}\text{Weavers} & 4:8\\ \text{Days} & 4:8\end{array}\right\}::4:x$$

$$\therefore\ 4\times 4\times x = 8\times 8\times 4 \quad\Rightarrow x=\frac{8\times 8\times 4}{(4\times 4)}=16$$

So, the required number of mats = 16.

20. (d): $z = 13 \times 1 + 12 = 25$,

$y = 9 \times z + 8 = 9 \times 25 + 8 = 233$,

$x = 5 \times y + 4 = 5 \times 233 + 4 = 1169$

$$\begin{array}{r|l}5 & x\\ \hline 9 & y-4\\ \hline 13 & z-8\\ \hline & 1-12\end{array}$$

Hence, on dividing 1169 by 585, remainder = 584.

MPP MONTHLY Practice Paper

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The solution of the equation $(3|x| - 3)^2 = |x| + 7$ which belongs to the domain of $\sqrt{x(x-3)}$ are given by

(a) $\pm\frac{1}{9}, \pm 2$ (b) $\frac{1}{9}, -3$

(c) $-\frac{1}{6}, -2$ (d) $-\frac{1}{9}, 0$

2. In a G.P. if the sum of infinite terms is 20 and the sum of their squares is 100, then the common ratio is

(a) 5 (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$

3. Let S be a non-empty subset of R. Consider the following statement:

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(a) There is a rational number $x \in S$ such that $x \leq 0$.

(b) There is no rational number $x \in S$ such that $x \leq 0$.

(c) Every rational number $x \in S$ satisfies $x \leq 0$.

(d) $x \in S$ and $x \leq 0$. $\Rightarrow$ x is not rational.

4. Six boys and six girls sit in a row randomly. The probability that the six girls sit together or the boys and girls sit alternately is

(a) $\frac{3}{308}$ (b) $\frac{1}{100}$ (c) $\frac{2}{205}$ (d) $\frac{4}{407}$

5. The general solution of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$ is

(a) $n\pi + \frac{\pi}{4}$ (b) $\frac{n\pi}{2} + \frac{\pi}{4}$

(c) $n\pi + \frac{\pi}{8}$ (d) $\frac{n\pi}{2} + \frac{\pi}{8}$

6. Let $P(n) = 3^{2n}$ $\forall$ $n \in N$, when divided by 8, leaves the remainder

(a) 2 (b) 1 (c) 4 (d) 7

One or More Than One Option(s) Correct Type

7. The real value of θ for which the expression $\frac{1+i\cos\theta}{1-2i\cos\theta}$ (where $i = \sqrt{-1}$) is a real number is

(a) $2n\pi + \frac{\pi}{2}, n \in I$ (b) $2n\pi - \frac{\pi}{2}, n \in I$

(c) $2n\pi \pm \frac{\pi}{2}, n \in I$ (d) $2n\pi \pm \frac{\pi}{4}, n \in I$

8. The number of ways of arranging seven persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order A-B-C-D (not necessarily together) is

(a) 210 (b) 5040

(c) $6 \times {}^7C_4$ (d) ${}^7P_3$

9. If n is a positive integer and $(3\sqrt{3}+5)^{2n+1} = \alpha + \beta$, where α is an integer and $0 < \beta < 1$, then

(a) α is an even integer

(b) $(\alpha + \beta)^2$ is divisible by 2^{2n+1}

(c) the integer just below $(3\sqrt{3}+5)^{2n+1}$ divisible by 3

(d) α is divisible by 10

10. If the lines $x - 2y - 6 = 0$, $3x + y - 4 = 0$ and $\lambda x + 4y + \lambda^2 = 0$ are concurrent, then

(a) $\lambda = 2$ (b) $\lambda = -3$
(c) $\lambda = 4$ (d) $\lambda = -4$

11. The equation $\sin x = [1 + \sin x] + [1 - \cos x]$ has (where $[x]$ is the greatest integer less than or equal to x)

(a) no solution in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(b) no solution in $\left[\frac{\pi}{2}, \pi\right]$

(c) no solution in $\left[\pi, \frac{3\pi}{2}\right]$

(d) no solution for $x \in R$

12. In a ΔABC $\tan A$ and $\tan B$ satisfy the inequation $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$, then

(a) $a^2 + b^2 + ab > c^2$ (b) $a^2 + b^2 - ab < c^2$
(c) $a^2 + b^2 > c^2$ (d) none of these

13. Let $f(x) = \dfrac{5\sqrt{\sin x}}{1 + \sqrt[3]{\sin x}}$. If D is the domain of f, then D contains

(a) $(0, \pi)$ (b) $(-2\pi, -\pi)$
(c) $(2\pi, 3\pi)$ (d) $(4\pi, 6\pi)$

Comprehension Type

ABC is a triangle right angled at A, $AB = 2AC$. $A = (1, 2)$, $B = (-3, 1)$. ACD is an equilateral triangle. The vertices of the two triangles are in anti-clockwise sense. $BCEF$ is a square with vertices in clockwise sense.

14. $\Delta ACF =$

(a) $\frac{51}{8}$ (b) $\frac{51}{4}$ (c) $\frac{31}{5}$ (d) $\frac{21}{4}$

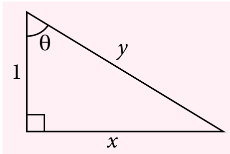
15. $DE =$

(a) $\sqrt{17}\sqrt{4-\sqrt{3}}$ (b) $\frac{\sqrt{17}}{2}\sqrt{8+\sqrt{3}}$

(c) $\frac{\sqrt{17}}{2}\sqrt{4+\sqrt{3}}$ (d) $\sqrt{15}\sqrt{4+\sqrt{3}}$

Matrix Match Type

16. A right angled triangle has sides 1 and x. The hypotenuse is y and the angle opposite to the side x is θ.



	Column I		Column II
P.	$\lim\limits_{\theta \to \frac{\pi}{2}} \sqrt{y} - \sqrt{x}$	1.	0
Q.	$\lim\limits_{\theta \to \frac{\pi}{2}} y - x$	2.	$\frac{1}{2}$
R.	$\lim\limits_{\theta \to \frac{\pi}{2}} y^2 - x^2$	3.	1
S.	$\lim\limits_{\theta \to \frac{\pi}{2}} y^3 - x^3$	4.	∞

	P	Q	R	S
(a)	4	3	2	1
(b)	1	1	3	4
(c)	1	2	3	4
(d)	2	2	4	3

Integer Answer Type

17. $\sin^2 12° + \sin^2 21° + \sin^2 39° + \sin^2 48° - \sin^2 9° - \sin^2 18°$ is

18. If α is a complex seventh root of unity, then the equation whose roots are $\alpha + \alpha^2 + \alpha^4$ and $\alpha^3 + \alpha^5 + \alpha^6$ is $x^2 + x + c = 0$, where c is

19. If the integer k is added to each of the numbers, 36, 300, 596, one obtains the squares of three consecutive terms of an A.P., then the last digit of k is

20. If $\log_{245} 175 = a$, $\log_{1715} 875 = b$, then the value of $\dfrac{1-ab}{a-b}$ is

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Keys are published in this issue. Search now! ☺

SELF CHECK

No. of questions attempted ......
No. of questions correct ......
Marks scored in percentage ......

Check your score! If your score is

> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
90-75%	GOOD WORK !	You can score good in the final exam.
74-60%	SATISFACTORY !	You need to score more next time.
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MPP MONTHLY Practice Paper

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer.

Self check table given at the end will help you to check your readiness.

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. Consider the following relations :
$R = \{(x, y) : x, y$ are real and $x = wy$ for some non-zero rational number $w\}$
$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) : m, n, p, q\right.$ are integers, such that $n, q \neq 0, mq = np\}$ then
(a) R is an equivalence relation, but S is not.
(b) S is an equivalence relation, but R is not.
(c) both R and S are equivalence relations.
(d) neither R nor S is an equivalence relation.

2. The points on the curve $y^3 + 3x^2 = 12y$ where tangent is vertical, are
(a) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (b) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$
(c) $(0, 0)$ (d) $\left(\pm\frac{11}{3}, 0\right)$

3. For the set of linear equations
$\lambda x - 3y + z = 0, x + \lambda y + 3z = 1, 3x + y + 5z = 2$
the value of λ, for which the equations does not have a unique solution.
(a) $-1, \frac{11}{5}$ (b) $-1, -\frac{11}{5}$ (c) $-\frac{11}{5}, 1$ (d) $1, \frac{11}{5}$

4. If from each of 3 boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn then the probability of drawing 2 white and 1 black ball is
(a) $\frac{13}{32}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{3}{16}$

5. $\sin^{-1}(\cos \sin^{-1} x) + \cos^{-1}(\sin \cos^{-1} x) =$
(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{4}$

6. If f is a continuous function and $\int_0^x f(t)dt \to \infty$ as as $|x| \to \infty$, then the number of points in which the line $y = mx$ intersects the curve $y^2 + \int_0^x f(t)dt = 2$ is
(a) 0 (b) 1 (c) atmost 1 (d) atleast 1

One or More Than One Option(s) Correct Type

7. Let $\int \frac{x^{1/2}}{\sqrt{(1-x^3)}}dx = \frac{2}{3} gof(x) + c$, then
(a) $f(x) = \sqrt{x}$ (b) $f(x) = x^{3/2}$
(c) $f(x) = x^{2/3}$ (d) $g(x) = \sin^{-1}x$

8. A normal is drawn at a point $P(x, y)$ of a curve. It meets the x-axis at Q. If PQ is of constant length k. Such a curve passing through $(0, k)$ is
(a) a circle with centre (0, 0)
(b) $x^2 + y^2 = k^2$
(c) $(1 - k)x^2 + y^2 = k^2$ (d) $x^2 + (1 + k^2)y^2 = k^2$

9. A student appears for tests I, II and III. The student is successful if he passes either in tests I and II or tests I and III. The probabilities of the student passing in tests I, II, III are p, q and 1/2 respectively. If the probability that the student is successful is 1/2, then
(a) $p = 1, q = 0$ (b) $p = 2/3, q = 1/2$
(c) $p = 3/5, q = 2/3$
(d) there are infinitely many values of p and q

10. The determinant
$$\Delta = \begin{vmatrix} b & c & b\lambda + c \\ c & d & c\lambda + d \\ b\lambda + c & c\lambda + d & a\lambda^3 + 3c\lambda \end{vmatrix}$$ is equal to zero, if

(a) b, c, d are in AP (b) b, c, d are in GP
(c) b, c, d are in HP
(d) λ is a root of $ax^3 - bx^2 + cx - d = 0$

11. If $P(2, 3, 1)$ is a point and $L \equiv x - y - z - 2 = 0$ is a plane then
(a) origin and P lie on the same side of the plane.
(b) distance of P from the plane is $\frac{4}{\sqrt{3}}$.
(c) foot of perpendicular is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$.
(d) image of point P by the plane is $\left(\frac{10}{3}, \frac{5}{3}, -\frac{1}{3}\right)$.

12. If $\phi(x) = f(x) + f(2a - x)$ and $f''(x) > 0, a > 0, 0 \le x \le 2a$, then
(a) $\phi(x)$ increases in $(a, 2a)$
(b) $\phi(x)$ increases in $(0, a)$
(c) $\phi(x)$ decreases in $(a, 2a)$
(d) $\phi(x)$ decreases in $(0, a)$

13. Which of the following functions are periodic?
(a) $f(x) = \sin x + |\sin x|$
(b) $g(x) = \frac{(1+\sin x)(1+\sec x)}{(1+\cos x)(1+\operatorname{cosec} x)}$
(c) $h(x) = \max(\sin x, \cos x)$
(d) $p(x) = [x] + \left[x + \frac{1}{3}\right] + \left[x + \frac{2}{3}\right] - 3x + 10$, where [.] denotes the greatest integer function.

Comprehension Type

Let A be the set of all 3×3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

14. The number of matrices in A is
(a) 12 (b) 6 (c) 9 (d) 3

15. The number of matrices A in $\mathscr{A}$ for which the system of equations $A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is inconsistent, is
(a) 0 (b) more than 2
(c) 2 (d) 1

Matrix Match Type

16.

	Column I	Column II	
P.	A line from the origin meets the lines $\frac{x-2}{1} = \frac{y-1}{-2} = \frac{z+1}{1}$, $\frac{x-8/3}{2} = \frac{y+3}{-1} = \frac{z-1}{1}$ at P and Q respectively. PQ^2 is	1.	7
Q.	The values of x satisfying $\tan^{-1}(x+3) - \tan^{-1}(x-3) = \sin^{-1}\frac{3}{5}$ are	2.	6
R.	Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a}\cdot\vec{b} = 0$, $(\vec{b}-\vec{a})\cdot(\vec{b}+\vec{c}) = 0$, $2\lvert\vec{b}+\vec{c}\rvert = \lvert\vec{b}-\vec{a}\rvert$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$, then the values of μ are	3.	5
S.	Let f be the function on $[-\pi, \pi]$ given by $f(0) = 9$ and $f(x) = \frac{\sin 9x/2}{\sin x/2}$ for $x \neq 0$. The value of $\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$ is	4.	4

	P	Q	R	S
(a)	2	4	3	4
(b)	3	4	2	1
(c)	3	2	1	4
(d)	1	2	3	4

Integer Answer Type

17. The distance between the skew lines $\frac{x}{4} = \frac{y-7}{-3} = \frac{z-7}{-2}$ and $\frac{x-4}{2} = \frac{y+6}{3} = \frac{z-34}{-10}$ is

18. The resultant of vectors $\vec{a}$ and $\vec{b}$ is $\vec{c}$. If $\vec{c}$ trisects the angle between $\vec{a}$ and $\vec{b}$ and $|\vec{a}| = 6, |\vec{b}| = 4$, then $|\vec{c}|$ is

19. Let $A = \{1, 2, 3, 4\}$ and $B = \{0, 1, 2, 3, 4, 5\}$. If m is the number of increasing functions from A to B and n is the number of onto functions from B to A, then the last digit of $n - m$ is

20. The area bounded by the y-axis, the tangent and normal to the curve $x + y = x^y$ at the point where the curve cuts the x-axis is

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SELF CHECK

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15 CHALLENGING PROBLEMS

For Entrance Exams

SECTION-I

SINGLE OPTION CORRECT

1. If $p_1, p_2, ..., p_n$ be probabilities of n mutually exclusive and exhaustive events $E_1, E_2, E_3, ..., E_n$ then $\left|\sum_{r=1}^{n} p_r \log_{(r+2)}(r+1)\right| \in$
(a) (0, 1) (b) (0, n) (c) (1, 2) (d) (1, n)

2. Sum of squares of all trigonometrical ratios has minimum value as
(a) 6 (b) 7
(c) 3 (d) None of these

3. The area bounded by $y = 2x \pm \sqrt{x^3}$, $x = 4$ is
(a) 160 sq. units (b) $\frac{128}{5}$ sq. units
(c) $\frac{4}{5}$ sq. units (d) None of these

4. If α_1, α_2 be the roots of $x^2 - 32x + 16 = 0$ and α_3, α_4 be the roots of $x^2 + 32x + 1 = 0$ then $\sum_{i=1}^{4} \alpha_i^2 =$
(a) 0 (b) 32
(c) 2013 (d) None of these

5. If e_1 and e_2 be the eccentricities of two concentric ellipses such that foci of one lie on the other and they have same length of major axes. If $e_1^2 + e_2^2 = p$ then
(a) $p < 1$ (b) $p > 1$ (c) $p \geq 1$ (d) $p \leq 1$

6. If $f(x) f(y) = f(x) + y f(y)$, where $f(x) \neq 0 \ \forall \ x \in N$ then $\underbrace{fofofo....f(1)}_{2013 \text{ times}} =$
(a) 2 (b) 2013
(c) 2014 (d) None of these

7. If $f(x) = \sqrt{36 - x^2} + \sin^{-1}\left(\frac{1}{2x} + \frac{x}{2}\right)$ and $g(x) = \left[\frac{f(x)}{R}\right]$ be continuous $\forall \ x \in R$ then least positive integral value of k is (where $[\cdot]$ denotes greatest integer function)
(a) 6 (b) 7
(c) 8 (d) None of these

8. In ΔABC, if $x = (\Sigma a)^2$, $y = \Sigma\, a^2$, $z = \Sigma \cot\frac{A}{2}$ and $w = \Sigma \cot A$ then
(a) $xy = zw$ (b) $xw = yz$
(c) $xz = yw$ (d) None of these

SECTION-II

MORE THAN ONE OPTION CORRECT

9. If $a = \sum_{r=1}^{\infty} \frac{1}{2r^2 - r}$, $b = \sum_{r=1}^{\infty} \frac{1}{2r^2 + r}$, $c = \sum_{r=1}^{\infty} \frac{1}{4r^3 - r}$ then which of the following is (are) correct?
(a) $a + b = 2$ (b) $b + c = 1$
(c) $a - c = 1$ (d) None of these

10. If $A = (1 + \omega)(1 + \omega^2)(1 + \omega^4) ... (1 + \omega^{2^{n-1}})$; $n \in N$, then $A =$
(a) 1, if n be even (b) $-\omega^2$, if n be even
(c) 1, for all $n \in N$ (d) $-\omega^2$, if n be odd

11. If $0 < \alpha_i < \frac{\pi}{3}$ $(i = 1, 2, 3, ..., n)$ and $\sum_{r=1}^{n} (z^{r-1} \sin\alpha_r) = \sqrt{3}$, then $|z| \in$
(a) $\left(\frac{1}{2}, \infty\right)$ (b) $\left(\frac{3}{4}, \frac{4}{5}\right)$
(c) $\left(\frac{3}{4}, 1\right)$ (d) None of these

12. If $y = \alpha x + a\beta$ be the common tangent to $y^2 = 4ax$ and $x^2 = 4by$ (where $a, b > 0$) then
(a) $\alpha\beta = 1$ (b) $\alpha < 0, \beta < 0$
(c) common tangent never passes through 1st quadrant
(d) None of these

13. If $f(x) = \dfrac{x^2+4x+3}{1+x+x^2} \in \left[\dfrac{a-b\sqrt{c}}{d}, \dfrac{a+b\sqrt{c}}{d}\right]$ (where a, b, d are integers and c prime) then which of the following are prime?
(a) $a + c$ (b) $a + d$ (c) $b + d$ (d) $c + d$

14. If $A = \begin{pmatrix} 5 & -3 \\ 111 & 336 \end{pmatrix}$ and
$\det(-3A^{2013} + A^{2014}) = \alpha^{\alpha}\beta^2(1 + \gamma + \gamma^2)$ then
(a) $\alpha = 2013$ (b) $\beta = 3$
(c) $\gamma = 10$ (d) None of these

15. From a point A on x-axis, 2 tangents are drawn to $x^2 + y^2 = 16$ meeting y-axis at P and Q then
(a) minimum value of $AP^2 + AQ^2$ is 128
(b) minimum area of ΔAPQ is 32 sq. units
(c) minimum value of $OA^2 + OP^2$ is 64
(d) for minimum area of ΔAPQ, the point A is $(4\sqrt{2}, 0)$ or $(-4\sqrt{2}, 0)$

SOLUTIONS

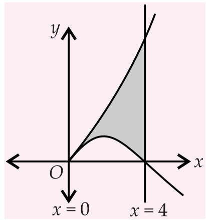
1. (a) : For mutually exclusive and exhaustive events, we have $p_1 + p_2 + p_3 + \ldots + p_n = 1$ ...(i)

$$\therefore \left|\sum_{r=1}^{n} p_r \log_{(r+2)}(r+1)\right| = \left|p_1\log_3 2 + p_2\log_4 3 + \ldots + p_n\log_{(n+2)}(n+1)\right|$$
$$\le |\log_3 2||p_1| + |\log_4 3||p_2| + \ldots + |\log_{(n+2)}(n+1)||p_n|$$
$$< |p_1| + |p_2| + |p_3| + \ldots + |p_n| \quad [\because \log_3 2 < 1 \text{ etc.}]$$
$$< p_1 + p_2 + p_3 + \ldots + p_n$$
$$< 1 \quad \text{[using (i)]}$$
$$\therefore \left|\sum_{r=1}^{n} p_r \log_{(r+2)}(r+1)\right| \in (0, 1)$$

2. (b) : $\because \sin^2\theta + \cos^2\theta + \sec^2\theta + \text{cosec}^2\theta + \tan^2\theta + \cot^2\theta$
$= 1 + (1 + \tan^2\theta) + (1 + \cot^2\theta) + \tan^2\theta + \cot^2\theta$
$= 3 + 2(\tan^2\theta + \cot^2\theta) = 3 + 4\left(\dfrac{\tan^2\theta + \cot^2\theta}{2}\right)$
$\ge 3 + 4\sqrt{\tan^2\theta\cot^2\theta}$ [$\because$ A.M. $\ge$ G.M.]
≥ 7
$\therefore$ Minimum value = 7.

3. (b) : $y = 2x + \sqrt{x^3}$ ...(i) $y = 2x - \sqrt{x^3}$ ...(ii)
$x = 4$ ...(iii)
(i) meets x-axis at $x = 0$ *i.e.* at (0, 0) only and then increases with x.

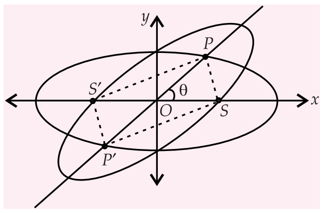
On solving (ii) with x-axis, we get
$x(2 - \sqrt{x}) = 0 \Rightarrow x = 0, 4$
Moreover, $y \to -\infty$ if $x \to \infty$



$\therefore$ Required area $= \displaystyle\int_0^4 (y_1 - y_2)dx$
(where y_1 and y_2 are values of y from (i) and (ii) respectively)
$$= \int_0^4 \{(2x + x^{3/2}) - (2x - x^{3/2})\}dx$$
$$= 2\int_0^4 x^{3/2}dx = 2\cdot\frac{2}{5}[x^{5/2}]_0^4 = \frac{4}{5}(32) = \frac{128}{5} \text{ sq. units}$$

4. (d) : $\alpha_1 + \alpha_2 = 32, \alpha_1\alpha_2 = 16$
$\alpha_3 + \alpha_4 = -32 ; \alpha_3\alpha_4 = 1$
$$\therefore \sum_{i=1}^{4}\alpha_i^2 = \alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2$$
$= (\alpha_1 + \alpha_2)^2 - 2\alpha_1\alpha_2 + (\alpha_3 + \alpha_4)^2 - 2\alpha_3\alpha_4$
$= 32^2 - 2(16) + (-32)^2 - 2\cdot 1$
$= 2(1024 - 16 - 1) = 2014$

5. (c): Let θ be the angle of inclination of major axes and equation of one of the ellipses be
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{...(i)}$$
whose foci are S and S' and eccentricity $= e_1$
$\therefore b^2 = a^2(1 - e_1^2)$ ...(ii)



Let P and P' be foci of other ellipse whose eccentricity $= e_2$
Let $PP' = 2R \Rightarrow OP = R = ae_2$
$\because$ Diagonals PP' and SS' bisect each other
$\therefore$ $PS'P'S$ is a parallelogram.
Here, co-ordinates of P are $(R\cos\theta, R\sin\theta)$
$$\therefore \text{From (i)}, \frac{R^2\cos^2\theta}{a^2} + \frac{R^2\sin^2\theta}{b^2} = 1$$
$$\Rightarrow \frac{a^2e_2^2(1 - \sin^2\theta)}{a^2} + \frac{a^2e_2^2\sin^2\theta}{a^2(1 - e_1^2)} = 1 \quad \text{[Using (ii)]}$$
$$\Rightarrow \sin^2\theta\left(\frac{e_2^2}{1 - e_1^2} - e_2^2\right) = 1 - e_2^2$$
$$\Rightarrow \sin^2\theta\left(\frac{e_1^2e_2^2}{1 - e_1^2}\right) = 1 - e_2^2$$

$$\Rightarrow \frac{(1-e_1^2)(1-e_2^2)}{e_1^2 e_2^2} = \sin^2\theta \le 1$$

$$\Rightarrow 1 - e_1^2 - e_2^2 + e_1^2 e_2^2 \le e_1^2 e_2^2$$

$$\Rightarrow e_1^2 + e_2^2 \ge 1 \Rightarrow p \ge 1$$

6. **(c):** $\because f(x)f(y) = f(x) + yf(y)$

$\therefore\ x = 1, y = 1 \Rightarrow f(1)f(1) = f(1) + 1\cdot f(1)$

$\Rightarrow f(1)f(1) = 2f(1)$

$\therefore\ f(1) = 2\ [\because f(x) \ne 0\ \forall\ x \in N\ \therefore f(1) \ne 0]$

$x = 2, y = 2 \Rightarrow f(2)f(2) = f(2) + 2f(2) = 3f(2)$

$\therefore\ f(2) = 3.$ [By same reason, $f(2) \ne 0$]

$x = 3,\ y = 3 \Rightarrow f(3)f(3) = f(3) + 3f(3) = 4f(3)$

$\therefore\ f(3) = 4$ etc.

$$\therefore\ \underbrace{fofofo....f(1)}_{2013\text{ times}} = \underbrace{fofofo....f(2)}_{2012\text{ times}} = \underbrace{fofofo....f(3)}_{2011\text{ times}}$$

$$= ...... = f(2013) = 2014$$

7. **(c):** $\because \sqrt{36 - x^2}$ is defined if $x^2 \le 36 \Rightarrow -6 \le x \le 6$

and, $\sin^{-1}\left(\frac{1}{2x} + \frac{x}{2}\right) = \sin^{-1}\left(\frac{1+x^2}{2x}\right)$ is defined if

$\left|\frac{1+x^2}{2x}\right| \le 1$ [taking $x \ne 0$]

i.e., if $\frac{1+x^2}{2|x|} \le 1 \Rightarrow 1 + x^2 \le 2|x|$

$$\Rightarrow \begin{cases} 1 + x^2 - 2x \le 0, & \text{if } x > 0 \\ 1 + x^2 + 2x \le 0, & \text{if } x < 0 \end{cases}$$

$$\Rightarrow \begin{cases} (1-x)^2 \le 0, & \text{if } x > 0 \\ (1+x)^2 \le 0, & \text{if } x < 0 \end{cases} \Rightarrow x = \pm 1$$

$$\Rightarrow D_f = \{-1, 1\} \Rightarrow f(-1) = \sqrt{35} - \frac{\pi}{2}$$

$$= 4.35 \text{ (approx.)}$$

$$f(1) = \sqrt{35} + \frac{\pi}{2} = 7.49 \text{ (approx.)}$$

$\therefore\ R_f = \{4.35, 7.49\}$ (approx.)

Now, $g(x) = \left[\frac{f(x)}{k}\right]$ will be continuous $\forall\ x \in R$

if $\frac{f(x)}{k} \in (0, 1)$ for which value of k must exceed the greatest value of $f(x)$ *i.e.*, 7.49

$\Rightarrow$ least positive integral value of $c = 8$.

8. **(b)** : $\because \cot\frac{A}{2} = \frac{2\cos^2\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{1+\cos A}{\sin A}$

$$= \frac{2bc(1+\cos A)}{2bc\sin A} = \frac{2bc + (b^2 + c^2 - a^2)}{4\Delta}$$

$$\therefore\ \Sigma\cot\frac{A}{2} = \frac{\Sigma 2bc + \Sigma(b^2 + c^2 - a^2)}{4\Delta}$$

$$= \frac{2(bc + ca + ab) + (a^2 + b^2 + c^2)}{4\Delta}$$

$$= \frac{(a+b+c)^2}{4\Delta} = \frac{(\Sigma a)^2}{4\Delta} = \frac{x}{4\Delta} \quad ...(i)$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{2bc\cos A}{2bc\sin A} = \frac{b^2 + c^2 - a^2}{4\Delta}$$

$$\Rightarrow \Sigma\cot A = \frac{\Sigma(b^2 + c^2 - a^2)}{4\Delta} = \frac{a^2 + b^2 + c^2}{4\Delta} = \frac{y}{4\Delta} \quad ...(ii)$$

On dividing (i) by (ii), $\frac{\Sigma\cot\frac{A}{2}}{\Sigma\cot A} = \frac{x}{y} \Rightarrow \frac{z}{w} = \frac{x}{y}$

$\therefore\ xw = yz$

9. **(a, b, c)** : $\because \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$ to ∞

Putting $x = 1$ on both sides, we get

$$\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ... \text{ to } \infty \quad ...(A)$$

$$= \frac{1}{1\cdot 2} + \frac{1}{3\cdot 4} + \frac{1}{5\cdot 6} + ... \text{ to } \infty$$

$$= \sum_{r=1}^{\infty} \frac{1}{(2r-1)2r} = \frac{1}{2}\sum_{r=1}^{\infty} \frac{1}{2r^2 - r} = \frac{a}{2}$$

$$\therefore\ a = 2\log_e 2 \quad ...(i)$$

From (i), $\log_e 2 = 1 - \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{1}{4} - \frac{1}{5}\right) - .....$ to ∞

$$= 1 - \frac{1}{2\cdot 3} - \frac{1}{4\cdot 5} - ... \text{ to } \infty \quad ...(B)$$

$$\Rightarrow \frac{1}{2\cdot 3} + \frac{1}{4\cdot 5} + \frac{1}{6\cdot 7} + ... \text{ to } \infty = 1 - \log_e 2$$

$$\Rightarrow \sum_{r=1}^{\infty} \frac{1}{2r(2r+1)} = 1 - \log_e 2$$

$$\Rightarrow \sum_{r=1}^{\infty} \frac{1}{2r^2 + r} = 2 - 2\log_e 2 \Rightarrow b = 2 - 2\log_e 2 \quad ...(ii)$$

Adding (A) and (B),

$$2\log_e 2 = 1 + \left(\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3}\right) + \left(\frac{1}{3\cdot 4} - \frac{1}{4\cdot 5}\right) + ... \text{ to } \infty$$

$$= 1 + \frac{2}{1\cdot 2\cdot 3} + \frac{2}{3\cdot 4\cdot 5} + ... \text{ to } \infty$$

$$\Rightarrow 2\log_e 2 - 1 = \sum_{r=1}^{\infty} \frac{2}{(2r-1)2r(2r+1)} = \sum_{r=1}^{\infty} \frac{1}{4r^3 - r}$$

$$\Rightarrow c = 2\log_e 2 - 1 \quad ... (iii)$$

$\therefore$ From (i), (ii) and (iii)

$a + b = 2,\ b + c = 1$ and $a - c = 1$

10. **(a, d)** : $A = (1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8) ... (1+\omega^{2^{n-1}})$

$$= (1+\omega)(1+\omega^{2^1})(1+\omega^{2^2})(1+\omega^{2^3})(1+\omega^{2^4}) ... \text{ to } n \text{ terms}$$

$$= (1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2) ... \text{ to } n \text{ terms}$$

$$= (-\omega^2)(-\omega)(-\omega^2)(-\omega) ... \text{ to } n \text{ terms}$$

$$= \begin{cases} 1, & \text{if } n \text{ be even} \\ -\omega^2, & \text{if } n \text{ be odd} \end{cases}$$

11. (b, c): $\because \sum_{r=1}^{n} z^{r-1}\sin\alpha_r = \sqrt{3}$

$\therefore\ \sin\alpha_1 + z\sin\alpha_2 + z^2\sin\alpha_3 + ... + z^{n-1}\cdot\sin\alpha_n = \sqrt{3}$

$\Rightarrow |\sin\alpha_1 + z\sin\alpha_2 + z^2\sin\alpha_3 + ... + z^{n-1}\cdot\sin\alpha_n| = |\sqrt{3}|$

$\Rightarrow\ \sqrt{3} \le |\sin\alpha_1| + |z\sin\alpha_2| + |z^2\sin\alpha_3| + ... + |z^{n-1}\sin\alpha_n|$

$\Rightarrow\ \sqrt{3} \le |\sin\alpha_1| + |z||\sin\alpha_2| + ... + |z^{n-1}||\sin\alpha_n|$

$< \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}|z| + \frac{\sqrt{3}}{2}|z|^2 + ...\text{to } \infty$

$\left[\because\ 0 < \alpha_i < \frac{\pi}{3}\ \therefore 0 < \sin\alpha_i < \frac{\sqrt{3}}{2}\right]$

$\Rightarrow\ \sqrt{3} < \frac{\sqrt{3}}{2}\{1 + |z| + |z|^2 + ...\text{ to } \infty\}$

$\Rightarrow\ 2 < \frac{1}{1-|z|}$ [Taking $0 < |z| < 1$]

$\Rightarrow\ 1 - |z| < \frac{1}{2}\ \Rightarrow\ |z| > \frac{1}{2}\quad \therefore |z| \in \left(\frac{1}{2}, 1\right)$

$\therefore$ Options (b) and (c) are correct.

12. (a, b, c) : $y^2 = 4ax$ .... (i) and $x^2 =$ ...(ii)

$\because\ y_1 = m_1x + \frac{a}{m_1}$ ... (iii) and $x = my + \frac{b}{m}$ ...(iv)

are any tangent to (i) and (ii) respectively.

From (iv) $y = \frac{1}{m}x - \frac{b}{m^2}$ ...(v)

If (iii) and (v) be same then $m_1 = \frac{1}{m}$ and $\frac{a}{m_1} = \frac{-b}{m^2}$

On eliminating m, we get $m_1^3 = -\frac{a}{b} \Rightarrow m_1 = \sqrt[3]{-\frac{a}{b}}$

$\therefore$ Common tangent is $y = \sqrt[3]{-\frac{a}{b}}\,x + a\cdot\sqrt[3]{-\frac{b}{a}}$

But, common tangent is given as $y = \alpha x + a\beta$

$\therefore\ \alpha = \sqrt[3]{-\frac{a}{b}}$ and $\beta = \sqrt[3]{-\frac{b}{a}}$

$\therefore\ \alpha < 0, \beta < 0$ and $\alpha\beta = 1$ $[\because\ a, b > 0]$

Further, common tangent can be written as

$\frac{x}{\left(-\frac{a\beta}{\alpha}\right)} + \frac{y}{a\beta} = 1$ in which intercepts on the axes are –ve.

$\therefore$ Common tangent will never pass through 1st quadrant.

13. (a, b, c) : Let $y = f(x) = \frac{x^2 + 4x + 3}{1 + x + x^2}$

$\therefore\ (y - 1)x^2 + (y - 4)x + (y - 3) = 0$

$\because\ x$ being real, $D \ge 0$ $[\because\ 1 + x + x^2 > 0\ \forall\ x \in R]$

$\Rightarrow\ (y - 4)^2 - 4(y - 1)(y - 3) \ge 0\ \Rightarrow\ 3y^2 - 8y - 4 \le 0$

$\Rightarrow\ y^2 - \frac{8y}{3} \le \frac{4}{3}\ \Rightarrow\ \left(y - \frac{4}{3}\right)^2 \le \frac{28}{9}$

$\Rightarrow\ \left(y - \frac{4}{3} + \frac{2\sqrt{7}}{3}\right)\left(y - \frac{4}{3} - \frac{2\sqrt{7}}{3}\right) \le 0$

$\Rightarrow\ y \in \left[\frac{4 - 2\sqrt{7}}{3}, \frac{4 + 2\sqrt{7}}{3}\right]$

$\therefore$ On comparing, we can write $a = 4$, $b = 2$, $c = 7$ and $d = 3$.

$\therefore\ a + c = 11$, $a + d = 7$ and $b + d = 5$ are primes.

14. (a, b, c) : $\because\ A = \begin{pmatrix} 5 & -3 \\ 111 & 336 \end{pmatrix}$

$\therefore\ |A| = 1680 + 333 = 2013$

$\therefore\ \det(-3A^{2013} + A^{2014}) = |A^{2013}(-3I + A)|$

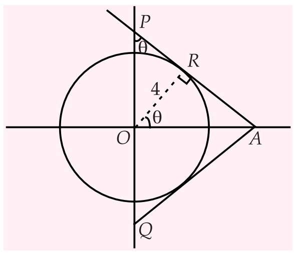
$= |A^{2013}||-3I + A| = 2013^{2013}\begin{vmatrix} 2 & -3 \\ 111 & 333 \end{vmatrix}$

$= 2013^{2013}(666 + 333) = 2013^{2013}(111 \times 9)$

$= 2013^{2013}\cdot 3^2(1 + 10 + 10^2)$

$\Rightarrow\ \alpha = 2013, \beta = 3, \gamma = 10$

15. (a, b, c, d) :



$\because\ \frac{OR}{OA} = \cos\theta\quad \therefore OA = 4\sec\theta$

And, $\frac{OR}{OP} = \sin\theta\quad \therefore OP = 4\operatorname{cosec}\theta$

$\therefore\ OA^2 + OP^2 = 16(\sec^2\theta + \operatorname{cosec}^2\theta)$

$= 16(2 + \tan^2\theta + \cot^2\theta)$

$= 32 + 32\left(\frac{\tan^2\theta + \cot^2\theta}{2}\right) \ge 32 + 32\sqrt{\tan^2\theta\cot^2\theta}$

[$\because$ A.M. $\ge$ G.M.]

$\therefore$ Minimum value of $OA^2 + OP^2 = 64$

Now, $AP^2 + AQ^2 = (OA^2 + OP^2) + (OA^2 + OQ^2)$

$= 2(OA^2 + OP^2) \ge 2(64)$ $[\because\ OP = OQ]$

$\therefore$ Minimum value of $AP^2 + AQ^2 = 128$

Also, $\Delta APQ = \frac{1}{2}\cdot PQ\cdot OA = \frac{1}{2}(2OP)\cdot 4\sec\theta$

$= 4\operatorname{cosec}\theta\cdot 4\sec\theta = \frac{32}{\sin 2\theta} \ge 32$ $[\because\ 0 < \sin 2\theta \le 1]$

$\therefore$ Minimum area of $\Delta APQ = 32$ sq. units when $\theta = 45°$

Then, $OA = 4\sec 45° = 4\sqrt{2}$

and so point A may be $(4\sqrt{2}, 0)$ or $(-4\sqrt{2}, 0)$

YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

1. There are four seats numbered 1, 2, 3, 4 in a room and four persons having tickets corresponding to these seats (one person having one ticket). Now the person having the ticket number 1, enters into the room and sits on any of the seats at random. Then the person having the ticket number 2, enters in room. If his seat is empty then he sits on his seat otherwise he sits on any of the empty seat at random. Similarly the other persons sit. Find the probability that the person having ticket numbered 4 gets the seat number 4. **– Pratiksha, Kolkata**

Ans. Way1 : 1 take 1, 2 take 2, 3 take 3, and 4 take 4

$$p\text{ (way 1)} = \frac{1}{4}\times 1\times 1\times 1 = \frac{1}{4}$$

Way 2 : 1 take 2, 2 take 1, 3 take 3, 4 take 4

$$p\text{(way 2)} = \frac{1}{4}\times\frac{1}{3}\times 1\times 1 = \frac{1}{12}$$

Way 3 : 1 take 2, 2 take 3, 3 take 1, 4 take 4

$$p\text{(way 3)} = \frac{1}{4}\times\frac{1}{3}\times\frac{1}{2}\times 1 = \frac{1}{24}$$

Way 4 : 1 take 3, 2 take 2, 3 take 1, 4 take 4

$$p\text{(way 4)} = \frac{1}{4}\times 1\times\frac{1}{2}\times 1 = \frac{1}{8}$$

$$\therefore \text{ Required probability} = \frac{1}{4}+\frac{1}{12}+\frac{1}{24}+\frac{1}{8} = \frac{1}{2}.$$

2. $P(x_1, y_1)$, $Q(x_2, y_2)$ with $y_1 < 0$ and $y_2 < 0$ be ends of latus rectum of the ellipse $\frac{x^2}{4}+y^2=1$. Find the equations of the parabola with P and Q as the ends of latus rectum. **– Md. Rizwan, Lucknow**

Ans. Ends of latus rectum are

$$\left(\sqrt{3}, \frac{1}{2}\right) \text{ and } \left(\sqrt{3}, -\frac{1}{2}\right), \left(-\sqrt{3}, \frac{1}{2}\right) \text{ and } \left(-\sqrt{3}, -\frac{1}{2}\right)$$

$$\therefore\ P = \left(\sqrt{3}, -\frac{1}{2}\right) \text{ and } Q = \left(-\sqrt{3}, -\frac{1}{2}\right)$$

Vertices are $A\left(0, \frac{-1+\sqrt{3}}{2}\right)$ or $A\left(0, \frac{-1-\sqrt{3}}{2}\right)$

$$\therefore \text{ Required equation} = x^2 - 2\sqrt{3}y = 3+\sqrt{3}$$

$$\text{or } x^2 + 2\sqrt{3}y = 3-\sqrt{3}.$$

3. $\hat{n}$ is a unit vector perpendicular to the plane containing the points whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{r}$, satisfying $|[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = \lambda|\vec{r}\times(\vec{b}-\vec{a})-\vec{a}\times\vec{b}|$ then find the value of λ.

– Arun Kumar, Patna

Ans. $(\vec{r}-\vec{a}) \times (\vec{b}-\vec{a})$ is parallel to $\hat{n}$

$\Rightarrow\ (\vec{r}-\vec{a}) \times (\vec{b}-\vec{a}) = t\hat{n}$ ...(i)

$\therefore\ |(\vec{r}-\vec{a}) \times (\vec{b}-\vec{a})\cdot\hat{n}| = |t|$

or $|[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = |t|$ ...(ii)

From (i), $\vec{r} \times (\vec{b}-\vec{a}) - \vec{a} \times \vec{b} = t\hat{n}$

or $|\vec{r}\times(\vec{b}-\vec{a}) - (\vec{a}\times\vec{b})| = |t|$ ...(iii)

From (ii) and (iii) we get,

$$|[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \hat{n}]| = |\vec{r}\times(\vec{b}-\vec{a}) - (\vec{a}\times\vec{b})|$$

$\therefore\ \lambda = 1.$

4. Prove that the roots of $1 + z + z^3 + z^4 = 0$ are represented by the vertices of an equilateral triangle. **– Manish Sharma, Punjab**

Ans. The given equation is $(1 + z)(1 + z^3) = 0$. The distinct roots being -1, $-\omega$, $-\omega^2$ which if be represented by points A, B and C in that order

$$AB = \sqrt{\left(\frac{3}{2}\right)^2+\frac{3}{4}} = \sqrt{3},\ BC = \sqrt{0+3} = \sqrt{3}$$

$$CA = \sqrt{\left(\frac{3}{2}\right)^2+\frac{3}{4}} = \sqrt{3}$$

The three points represent the vertices of an equilateral triangle.

Solution Sender of Maths Musing

SET-170

1. Ravinder Gajula (Karimnagar)
2. Satyadev (Bangalore)

SET-171

1. N. Jayanthi (Hyderabad)
2. Khokon Kumar Nandi (West Bengal)
3. V. Damodhar Reddy (Telangana)

MATHS MUSING

SOLUTION SET-171

1. (b) : Using the polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$,

$xdx + ydy = \sqrt{x^2 + y^2}\, dx$ becomes

$dr(1 - \cos\theta) + r\sin\theta d\theta = 0$

$\Rightarrow r(1 - \cos\theta) = c$, $r - x = c$, (where c = constant)

$x^2 + y^2 = r^2 = (c + x)^2 \Rightarrow y^2 = 2cx + c^2$, family of parabolas.

2. (b) :

$$\frac{\cos A + \cos C}{a + c} + \frac{\cos B}{b} = \frac{b\cos A + b\cos C + a\cos B + c\cos B}{b(a+c)}$$

$$= \frac{(b\cos A + a\cos B) + (b\cos C + c\cos B)}{b(a+c)}$$

$$= \frac{a+c}{b(a+c)} \quad [\because c = b\cos A + a\cos B \text{ and } a = b\cos C + c\cos B]$$

$$= \frac{1}{b}$$

3. (b) : $\begin{vmatrix} 4-\lambda & 6 & 6 \\ 1 & 3-\lambda & 2 \\ -1 & -4 & -3-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^3 - 4\lambda^2 - \lambda + 4 = 0$

$\therefore A^3 - 4A^2 - A + 4I = 0$; Multiplying by A^{-1}, we get

$$A^{-1} = -\frac{1}{4}\cdot A^2 + A + \frac{1}{4}I$$

$$\alpha = -\frac{1}{4}, \beta = 1, \gamma = \frac{1}{4}, \alpha + \beta + \gamma \neq 1$$

4. (c) : Let $C = \frac{\pi}{2}$ in the triangle ABC.

$$r = \frac{ab}{a + b + \sqrt{a^2 + b^2}}$$

$\Rightarrow r\sqrt{a^2 + b^2} = ab - r(a + b)$

Squaring both sides, we get

$(a - 2r)(b - 2r) = 2r^2$

The number of triangles is half of the number of divisors of $2r^2 = 2 \cdot 3^2 \cdot 11^2 \cdot 61^2$, which is $\frac{1}{2} \times 2 \times 3^3 = 27$

5. (d) : $A = (0, -1)$ and $P = (2\cos\theta, \sin\theta)$

$AP^2 = 4\cos^2\theta + (\sin\theta + 1)^2 = 5 - 3\sin^2\theta + 2\sin\theta$

$$= \frac{16}{3} - 3\left(\sin\theta - \frac{1}{3}\right)^2 \leq \frac{16}{3} \quad \therefore \quad AP_{\max.} = \frac{4}{\sqrt{3}}$$

6. (d) : $\vec{r} \times \hat{i} = \hat{j} + \hat{k} \Rightarrow \vec{r} = -\hat{j} + \hat{k} + \alpha\hat{i}$

$\vec{r} \times \hat{j} = \hat{k} + \hat{i} \Rightarrow \vec{r} = \hat{i} - \hat{k} + \beta\hat{j}$

The lines $\vec{r} = \vec{a} + \alpha\vec{b}$ and $\vec{r} = \vec{c} + \beta\vec{d}$ are skew if $[\vec{c} - \vec{a} \;\; \vec{b} \;\; \vec{d}] \neq 0$

Here, $[\hat{i} + \hat{j} - 2\hat{k} \;\; \hat{i} \;\; \hat{j}] = -2$

$\therefore$ The lines are skew.

7. (d) : $A = (-1, 2)$, $B = (2, 3)$

$\Rightarrow$ Line AB is $x - 3y + 7 = 0$

The circle is $(x + 1)(x - 2) + (y - 2)(y - 3) + \lambda(x - 3y + 7) = 0$

or $S = x^2 + y^2 + (\lambda - 1)x - (3\lambda + 5)y + 7\lambda + 4 = 0$ ...(i)

$$\text{Radius} = \sqrt{5} \Rightarrow \frac{(\lambda - 1)^2 + (3\lambda + 5)^2}{4} - (7\lambda + 4) = 5$$

$\Rightarrow \lambda^2 = 1, \lambda = 1, -1$

$\lambda = 1 \Rightarrow S$ is $x^2 + y^2 - 8y + 11 = 0$. It does not intersect x-axis.

$\lambda = -1 \Rightarrow S$ is $x^2 + y^2 - 2x - 2y - 3 = 0$

$\therefore y = 0 \Rightarrow x = -1, 3$

Length of intercept is $3 + 1 = 4$.

8. (a) : Since,

$S = x^2 + y^2 + (\lambda - 1)x - (3\lambda + 5)y + 7\lambda + 4 = 0$ ...(i)

Putting $y = -x$ in eq. (i)

$2x^2 + 4(\lambda + 1)x + 4 + 7\lambda = 0$

It has equal roots $\Rightarrow \lambda = 2, -\frac{1}{2}$.

$\lambda = 2, y = 0 \Rightarrow$ (i) becomes $x^2 + x + 18 = 0$,

$\therefore$ S does not intersect x-axis.

$\lambda = -\frac{1}{2}, y = 0 \Rightarrow$ (i) becomes $2x^2 - 3x + 1 = 0$

$\Rightarrow x = 1, \frac{1}{2}$

Length of x-intercept $= 1 - \frac{1}{2} = \frac{1}{2}$.

9. (1) : In the given equation, $\alpha + \beta = p$, $\alpha\beta = -(p + q)$

$$\text{The given expression is } \frac{(\alpha+1)^2}{(\alpha+1)^2 + q - 1} + \frac{(\beta+1)^2}{(\beta+1)^2 + q - 1}$$

$$= \frac{\alpha + 1}{\alpha + 1 - (\beta + 1)} + \frac{\beta + 1}{\beta + 1 - (\alpha + 1)} = 1$$

Since, $q - 1 = -(\alpha\beta + \alpha + \beta + 1) = -(\alpha + 1)(\beta + 1)$

10. (c) : $f_1'(x) = \frac{\cos x}{|\cos x|}$, $f_2'(x) = -\frac{\sin x}{|\sin x|}$

$f_3'(x) = \frac{-\cos x}{|\cos x|}$, $f_4'(x) = \frac{\sin x}{|\sin x|}$

$3 \in \left(\frac{\pi}{2}, \pi\right) \Rightarrow$ the derivaties are $-1, -1, 1, 1$

$4 \in \left(\pi, \frac{3\pi}{2}\right) \Rightarrow$ the derivatives are $-1, 1, 1, -1$

$5 \in \left(\frac{3\pi}{2}, 2\pi\right) \Rightarrow$ the derivatives are $1, 1, -1, -1$

$7 \in \left(2\pi, \frac{5\pi}{2}\right) \Rightarrow$ the derivatives are $1, -1, -1, 1$